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Optimal Order Quantity For The Retailer Under Different Quantity Discount Of The Supplier

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Abstract

In the bilateral negotiation literature, an increasing body of work studies how buyers can make multi-issue trade-off to maximize its own expected profit. Various multi-issue negotiation models for such incentive schemes have been proposed, but a critical research ignored in this research field is that the quantity discount made by sellers could influence the order quantity of buyers. In this paper, we propose a novel bilateral two-issue negotiation model that price and quantity are negotiated. Suppliers offer the product at a given price and apply discounts according to the quantity ordered. We derive the optimal order quantity for the retailer under different quantity discount of the supplier. A numerical example is provided that illustrate the solution procedure.

Keywords: optimal, order quantity, quantity discount, bilateral negotiation.

1. Introduction

Bilateral negotiation is the most prevalent one in all sorts of negotiation problems. Alternating-offer negotiation protocol is the most predominant way for solving bilateral negotiation problem. Most researchers analyzed this problem between economics and artificial intelligence. The pioneering work about alternating-offer negotiation in economics field is Rubinstein's alternating-offer bargaining game [1]. In this model, perfect information is assumed and takes time into consideration. This model has been applied to automated negotiation in artificial intelligence widely, since there has a unique solution in this work [2]. Faratin et al. [3] assumed both sides have limited knowledge and computational resources, and designed the reasoning mechanisms in a service-oriented negotiation. The negotiation framework they proposed considers the agent's time deadlines and does not make the assumption that both sides have perfect information. Ren and Zhang [4] presented a bilateral single-issue negotiation model considering time constraint and nonlinear utility. Zhang and Chen [5] presented a sealed-bid single-issue negotiation model in which both agents simultaneously submit offers instead of alternating offer by introducing a mediator agent.

However, multi-issue is often involved in the bilateral negotiation. For example, both sides may need to reach an agreement about the product that is characterized by some issues such as price, quantity, delivery time, etc. There is an increasing body of work that studies how both sides can make multi-issue trade-off to maximize respective expected utility. Fatima et al. [6] proposed an agenda-based framework and investigated the negotiation outcomes in an incomplete information setting. Chen and Pu[7] proposed a multi-issue negotiation model and indicated that nonlinear preference elicitation is a time consuming

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process or sometimes be intractable. Louta et al. [8] proposed some specific negotiation strategies that the agents prepare bids for and evaluate offers on behalf of the users aiming to maximize their owner's utility. Zhang et al. [9] presented a multi-issue negotiation model in which both agents simultaneously submit offers by introducing a mediator agent.

These studies have dealt with negotiation problems involving multi-issue. However, most existing researches in negotiation area don't take into account the relation between price and quantity. There are also some researchers analyze how the quantity discount influence the price, and give the optimal quantity discount[10-14]. Ref[10] use the fuzzy rough to construct a economic order quantity model and considering quantity discount and prepayment. Ref [14] presented the supplier's optimal quantity discount policy consider asymmetric information. However, these studies almost analyzed the optimal quantity discount from the perspective of the supplier, and ignored the analysis of optimal order quantity for the retailer.

In this paper, we focus on an analysis of the optimal order quantity for the retailer, under different quantity discount of the supplier. We analyze the optimal order quantity of the retailer aiming to minimize its expected loss based on three kinds of situations. The remainder of this paper is organized in the following manner. Section 2 presents our general negotiation model about the retailer and the supplier. In Section 3, we analyze the optimal order quantity for the retailer under different quantity discount of the supplier, which minimize its expected loss. Section 4 demonstrates the solution procedure by a numerical example. Finally, in Section 5, we draw the conclusion and outline some directions for future plans.

2. The negotiation model

In this model, we assume the retailer is a self-employed businessmen. i.e., he is a retailer. The retailer and supplier negotiate on the quantity of a certain good. The price of the good is firm and generally not negotiable. Let p denotes the price of the good. The retailer does not enjoy any price discount when the order quantity q is less than a certain quantity Q_1. However, it will enjoy a discount rate α only if the order quantity is more than a certain quantity Q_1. The retailer determines its own order quantity q according to its estimate of the market demand. It can be assumed the retailer considers the market demand x is uniformly distributed in a certain interval $[a,b]$. Therefore, the order quantity q of the retailer should also be located in the interval $[a,b]$. In reality, if the order quantity is larger than the actual market demand, the retailer will dispose the goods at a certain loss. For example, the supplier promised the retailer that the goods could be returned within a stated time, so long as the retailer is responsible for all further shipping fees. So, we assume that the loss is a fixed charge F. let p_0 denotes the sale price of the retailer.

3. Optimal order quantity of the retailer

We analyze the optimal order quantity of the retailer aiming to minimize its expected loss. Three kinds of situations will be discussed below.

3.1 The first situation where $Q_1 \le a$

In this case, the retailer will enjoy a discount rate α , i.e., the purchase price of the good is αp . It can be specified the retailer's loss function f_1 as follows:

$$f_1 = \begin{cases} (x-q)(p_0 - \alpha p) & if\ q \le x \le b \\ F & if\ a \le x < q \end{cases}.$$

Note x is a variable which is uniformly distributed in the interval $[a, b]$. Then we could calculate the retailer's expected loss function L_1 for a quantity q as follows:

$$\begin{aligned} L_1(q) &= \int_a^b f_1 \cdot \frac{1}{b-a} dx \\ &= \frac{1}{b-a} \cdot \left(\int_a^q F dx + \int_q^b (x-q)(p_0 - \alpha p) dx \right) \\ &= \frac{1}{b-a} \cdot \left(F(q-a) + \frac{(p_0 - \alpha p)(b-q)^2}{2} \right) \end{aligned} \tag{1}$$

3.2 The second situation where $Q_1 > b$

In this case, the retailer does not enjoy any price discount, i.e., the purchase price of the good is p. We can specify the retailer's loss function f_2 as follows:

$$f_2 = \begin{cases} (x-q)(p_0 - p) & if\ q \le x \le b \\ F & if\ a \le x < q \end{cases}.$$

Similarly, we could calculate the retailer's expected loss function L_2 for a quantity q as follows:

$$\begin{aligned} L_2(q) &= \int_a^b f_2 \cdot \frac{1}{b-a} dx \\ &= \frac{1}{b-a} \cdot \left(F(q-a) + \frac{(p_0 - p)(b-q)^2}{2} \right) \end{aligned} \tag{2}$$

3.3 The third situation where $a < Q_1 \le b$

In this case, two kinds of situations are discussed respectively.

1) $a < x \le Q_1$; In this case, the retailer doe not enjoy any price discount when q is less than x. However, the retailer will be responsible for a certain loss F when q is more than x. We can specify the retailer's loss function f_{31} as follows:

$$f_{31} = \begin{cases} (x-q)(p_0 - p) & if\ a < q \le x \le Q_1 \\ F & if\ a < x < q \le Q_1 \end{cases}.$$

Then we could calculate the retailer's expected loss function L_{31} for a quantity q as follows:

$$L_{31}(q)=\int_a^{Q_1} f_{31}\cdot\frac{1}{Q_1-a}\,dx$$
$$=\frac{1}{Q_1-a}\cdot\left(F(q-a)+\frac{(p_0-p)(Q_1-q)^2}{2}\right)$$

(3)

2) $Q_1 < x \le b$; In this case, the retailer does not enjoy any price discount when q is less than Q_1. Otherwise, the retailer will enjoy a discount rate α. We can specify the retailer's loss function f_{32} as follows:

$$f_{32}=\begin{cases}(x-q)(p_0-\alpha p) & if\ Q_1<q\le x\le b\\ F & if\ Q_1<x<q\le b\\ (x-q)(p_0-p) & if\ q\le Q_1\end{cases}.$$

Similarly, we could calculate the retailer's expected loss function L_{32} for a quantity q as follows:

$$L_{32}(q)=\frac{1}{b-Q_1}\left(\int_{Q_1}^{q} F\,dx+\int_q^b (x-q)(p_0-\alpha p)\,dx\right)+\frac{(x-q)(p_0-p)}{Q_1-a} \tag{4}$$

In conclusion, if $a<Q_1\le b$, then we could calculate the retailer's expected loss function L_3 as follows:

$$L_3(q)=\frac{1}{b-a}\left(\int_a^{Q_1} L_{31}\cdot dx+\int_{Q_1}^{b} L_{32}\cdot dx\right)$$
$$=\frac{1}{b-a}\left(F(2q-a-Q_1)+\frac{(p_0-p)(Q_1-q)^2+(p_0-\alpha p)(b-q)^2}{2}+\frac{(p_0-p)(b-Q_1)(b+Q_1-q)}{2(Q_1-a)}\right) \tag{5}$$

3.4 Main propositions

Proposition 1. When there is $Q_1 \le a$, the retailer's optimal order quantity q satisfies the following equations: $q_1^*=b-\frac{F}{p_0-\alpha p}$; and the retailer's minimum expected loss function L_1 for a quantity q is set as the following equation: $L_1^*=F-\frac{F^2}{2(b-a)(p_0-\alpha p)}$.

Proof. From Eq (1), we could take the derivative of function $L_1(q)$ with respect to q and set the derivation to zero: $\frac{d\,L_1(q)}{d\,q}=\frac{F-(b-q)(p_0-\alpha p)}{b-a}=0$. We could obtain the value of q as follows: $q=b-\frac{F}{p_0-\alpha p}$. Taking the second derivative of function $L_1(q)$ with respect to q, we have: $\frac{d^2\,L_1(q)}{d\,q^2}=\frac{1}{b-a}\ge 0$. Then $q=b-\frac{F}{p_0-\alpha p}$ minimizes $L_1(q)$. In this case, the value of function $L_1(q)$

is $F-\dfrac{F^2}{2(b-a)(p_0-\alpha p)}$ .Thus, when $q=b-\dfrac{F}{p_0-\alpha p}$, the function $L_1(q)$ achieves the minimum $F-\dfrac{F^2}{2(b-a)(p_0-\alpha p)}$.

In conclusion, we could obtain the above proposition. □

Proposition 2. When there is $Q_1 > b$, the retailer's optimal order quantity q satisfies the following equations: $q_2^* = b-\dfrac{F}{p_0-p}$; and the retailer's minimum expected loss function L_2 for a quantity q is set as the following equation: $L_2^* = F-\dfrac{F^2}{2(b-a)(p_0-p)}$.

Proof. From Eq (2), we could take the derivative of function $L_2(q)$ with respect to q and set the derivation to zero: $\dfrac{d\,L_2(q)}{d\,q}=\dfrac{F-(b-q)(p_0-p)}{b-a}=0$.

We could obtain the value of q as follows: $q=b-\dfrac{F}{p_0-p}$. Taking the second derivative of function $L_2(q)$ with respect to q, we have: $\dfrac{d^2\,L_2(q)}{d\,q^2}=\dfrac{1}{b-a}\ge 0$. Then $q=b-\dfrac{F}{p_0-p}$ minimizes $L_2(q)$. In this case, the value of function $L_2(q)$ is $F-\dfrac{F^2}{2(b-a)(p_0-p)}$.

Thus, when $q=b-\dfrac{F}{p_0-p}$, the function $L_2(q)$ achieves the minimum $F-\dfrac{F^2}{2(b-a)(p_0-p)}$.

In conclusion, we could obtain the above proposition. □

Proposition 3. When there is $a < Q_1 \le b$, the retailer's optimal order quantity q satisfies the following equations:

$$q_3^*=\frac{(p_0-p)Q_1+(p_0-\alpha p)b-2F+{(p_0-p)(b-Q_1)}\big/{2(Q_1-a)}}{2p_0+(1+\alpha)p}.$$

Proof. From Eq (5), we could take the derivative of function $L_3(q)$ with respect to q and set the derivation to zero:

$$\frac{d\,L_3(q)}{d\,q}=\frac{2F-(p_0-p)(Q_1-q)\text{-}(p_0-\alpha p)(b-q)-{(p_0-p)(b-Q_1)}\big/{2(Q_1-a)}}{b-a}=0.$$

We could obtain the value of q as follows:

$$q=\frac{(p_0-p)Q_1+(p_0-\alpha p)b-2F+{(p_0-p)(b-Q_1)}\big/{2(Q_1-a)}}{2p_0+(1+\alpha)p}.$$

Taking the second derivative of function $L_3(q)$ with respect to q, we have: $\dfrac{d^2\,L_3(q)}{d\,q^2}=\dfrac{2p_0-(1+\alpha)p}{b-a}$. It can be seen easily that $\dfrac{d^2\,L_3(q)}{d\,q^2}\ge 0$.

Thus, when $q = \dfrac{(p_0 - p)Q_1 + (p_0 - \alpha p)b - 2F + \dfrac{(p_0 - p)(b - Q_1)}{2(Q_1 - a)}}{2p_0 + (1+\alpha)p}$, the function $L_3(q)$ achieves the minimum.

In conclusion, we could obtain the above proposition. □

4. Numerical analysis and discussion

In this section, the proposed negotiation procedure is applied to an illustrative test case.

4.1 Setting

The simulated e-market is characterized by bilateral negotiation between a retailer and a supplier. We consider a Retailer B and a Supplier S that negotiates over a specific product described by two issues: price p, quantity q. The price of the good is firm and generally not negotiable, i.e., $p = 10$; but the retailer could enjoy a certain price discount α where $\alpha = 0.7$ when the order quantity q reaches a certain number Q_1. The retailer determines its own order quantity q according to its estimate of the market demand x. it can be assumed the market demand d is uniformly distributed between a and b where $a = 50, b = 200$. When the order quantity is larger than the actual market demand, the retailer will dispose the goods at a fixed charge F where $F = 40$. The sale price of the retailer is as follows: $p_0 = 18$.

1) $Q_1 = 30$; According to proposition 1, we could calculate the optimal order quantity of the retailer is q^*=196 ;

2) $Q_1 = 200$; According to proposition 2, we could calculate the optimal order quantity of the retailer is q^*=195 ;

3) $Q_1 = 150$; According to proposition 3, we could calculate the optimal order quantity of the retailer is q^*=63 .

4.2 Discussion

From the above numerical analysis, it can be seen that the optimal order quantity of the retailer is not obviously impacted by the quantity discount of the supplier when the discount quantity is not included in the filed of the retailer's estimate for the market demand. However, when the discount quantity lies in the interval of the retailer's estimate for the market demand, the optimal quantity of the retailer is obviously difference compared with the former two situations.

5. Conclusions

In this paper, we focus on an analysis of the optimal order quantity of the retailer aiming to minimize its expected loss, based on three different quantity discounts of the supplier. The bilateral negotiation model presented in this paper considers the interactive relation between price and quantity. Moreover, we analyze the optimal order quantity based on an assumption that the quantity discount of the supplier is only two situations. Our aim for the future is to extend this model to more situations of the quantity discount for the supplier. The interactive relation between the quantity discount of the supplier and the order quantity of the retailer will be further studied.

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