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The Research on Congestion Rate of Single Item Selection System in Wide Aisles under Equivalent Picking and Walking Speed

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Abstract

It constructs Markov model for the blocking rate of wide aisles picking system under equal picking and walking speed. For describing the status of the transfer, there are innovative ideas. Get the analytical expression of picking area, picking density and blocking rate. We analyze various factors on the impact of blocking rate. The conclusion is that the larger picking area, the lower blocking rate, and within a certain range picking density, the blocking reach the maximum.

Key words: Blocking, Order picking, Wide aisles, Markov

1. Introduction

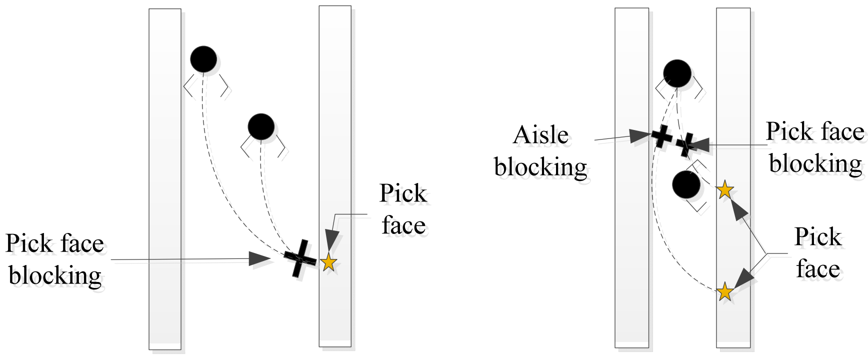
Order picking is based on customers' order requirements in a distribution center, then seized goods from a storage location quickly and accurately, and do some classification, concentrated, assembly according to a certain way. Improving materials handling equipment makes the picking paths become narrow. And the congestion between the pickers will reduce the operating efficiency, so it is needs to increase labor costs to ensure the original efficiency. Some enterprises adopt wide aisle strategy, but there would still be pick face blocking problems， which result in some picker to wait a short time and impact the operating efficiency of the picking system.

Picking blocking is the bottleneck for affecting working efficiency in a picking system. It is an effective way to improve customer satisfaction, reduce logistics costs and improve supply chain service levels. It overcomes picking blocking and improves picking efficiency. From our literature search, we observe that scholars from various countries using different methods studied on storage area configuration, storage policy, picking strategy and path policy of order picking time and cost from different perspectives, but the study of picking blocking formed during picking is still relatively small.

Gu (2010) [1] consider the warehouse layout design can influence the future optimize operation and made a significant impact on the order picking and picker's picking path. Roodbergen (2006) [2]build a model that in S-type and maximum spacing method path policy conditions that minimize the picking path by the aisle length, number, pick position, and ultimately concluded that the selection of picking path policy has a certain extent affect the warehouse size and layout. Kevin (2012) [3] describes the study on optimizing on fishbone warehouse, and optimized structure will reduce the walking route 10%-15% less than traditional in a multi-constraint. ZHOU (2014) [4] considers that the desired picking distance of fishbone layout is smaller than traditional layout, but needs to add more storage space to ensure that the same number of storage spaces. Gue and Meller(2006) [5] studied congestion of the annular shelf layout of the narrow aisles. Their study shows that in the narrow aisles, the picker will meet congestion within aisles and more jam with the increase in the number of pickers, lighten with increase of the picking zone, picker walking speed quicker cause more serious congestion. Skufca (2005) [6] has established a model of more pickers congestion in the narrow aisles system, which is based on the assumption of unlimited walking speed and single item picking system. Pratik and Meller(2009) [7] studied on congestion between the two pickers of the annular shelf layout of the wide aisles. They believe that congestion in a wide aisles layout than the narrow aisles has eased. Congestion is an outstanding problem in the high turnover and multi pickers system. Soondo Hong(2013) [8] further verified the results of Gue and Pratik, and has made the improvement in the congestion estimation algorithm.

2. Generation of congestion

When there are multiple pickers working at the same time, blocking maybe occur. When the distance between two pickers is zero, that is, the blocking occurs. Blocking phenomenon is divided into two kinds. One is pick face blocking which easier occur in wide and narrow aisles, and the other is the aisle blocking, easier occur in narrow aisles, as shown in figure 1.



(a) Blocking in wide aisles (b) Blocking in narrow aisles

Fig. 1 Aisle blocking and pick face blocking in wide and narrow aisles layout

In wide aisles layout, when the picking state of the upper picker is being picking, the downstream picker is waiting for picking at the same operation pick face, and the distance between them is zero, blocking occurs. Therefore, the layout of a wide aisle, only one kind of blocking occurs, that is pick face blocking.

In this paper, we study the blocking problem of the picking system using the traditional rectangle layout and S-type path strategy. Because of the change of the distance between the pickers with no aftereffect characteristics, it is suitable for establishing the Markov model for distance variation among the pickers, and the validity of the model is verified by simulation and analysis of the factors affecting the blocking.

3. Model hypothesis and symbol description

3.1 Description of Wide aisles picking system

Because of the variety of the picking system, and with realistic complexity, in order to describe the theoretical model simple, in this paper, we study the traditional shelf layout of wide aisles containing n picking faces. More than one picker is picking under the random storage and S-type path strategy. In one unit time period, the probability of the picking of the picker is p, walking probability is q. In the single picking system that one can only pick one item at a picking face. If one is picking at this time period, then the probability that he will walk to the next picking face at the next period is 1.

In this paper, we construct a blocking model under the equal picking and walking speed, that is to say, the time of walking is equal to the time of picking when pass a picking face. Expressed as the time ratio of picking and walking is $P:W=1:1$.

3.2 Symbol description

In the traditional shelf layout system to construct blocking model, make the following explanation of symbols.

(1)The order picking region has n picking faces. In the standard case, a picking face is a column of the tray shelf or a group of fluent shelves. In fact, each picking face may contain several picking location, for example, a column tray shelf may contain 3 or 4 layers of the tray rack. In this paper, the picking position of the picker is picking face.

(2)In this paper, wide aisle picking system is studied. The picker can be a person, or an automatic picking. In wide aisles, more than one picker can be accommodated to picking and passing. But it can only accommodate one picker operating in a pick face. Therefore, it only can generate surface blocking.

(3) The person follows the path of the S-type path strategy, which means that the picker make the picking only in one direction in the aisles.

(4) In the actual distribution center, there are two rows of shelves in each aisle, in order to simplify the problem, suppose that the picking is not associated with any of the two shelves in the aisles.

(5) The states of the pickers in the picking zone is picking, walking or blocking.

(6) The probability that the picker picks at each picking face is p . The probability of not picking is $q=1-p$. If one picker picks $I(I \le n)$ goods in average, the probability of one goods picked on a certain picking face is $p=I/n$ in a single picking system.

3.3 State description

Denote states of two pickers during *t* period is S_i^t , and subscript *i* shows possible types of states. We also respectively denote positions of two pickers in terms is N_1 and N_2 , and the distance between two pickers in terms of d_{12}^t when *t* period ends. So we have $d_{12}^t=(n+N_1\text{-}N_2)\bmod n$, and the specific expression of S_i^t are $d_{pp},d_{pw},d_{wp},d_{ww},(d=0,1,\ldots,n-1)$, which respectively expresses the distance and state between two pickers when *t* period ends (here omitted superscript *t*) , and the first and second subscript respectively expresses the state of picker 1 and picker 2. For example, when $d_{12}=0$, 0_{pp} means the distance between two pickers is zero and both of them are in picking states. So, 0_{pp} is blocking state. Once blocking occurs, the downstream picker must wait for the upstream picker to finish work at the same operation pick face.

4 Markov model under picking and walking speed ratio is 1:1

4.1 State transition matrix

In this paper, we study the traditional shelf layout have n picking faces, and the situation of two pickers are picking at the same time in the wide aisles. In the single item picking system, when picking and walking speed ratio is 1:1, that means the required time of picking one item from a picking face, is equal to required time of walking through a picking face.

Below, we analysis the case that the status transition of the two pickers from $t=0$ to $t=1$. Figure 3 shows the state transfer from $t=0$ with state 0_{pp}, where 0_{pp} is a congested state, till the next period $t=1$, picker 1 will walk with the probability of 1. The upper picker of may be any of picker 1 or picker 2.

When the status is 0_{pw}, the distance between two pickers is one pick face. At the moment, picker 1 is picking, and picker 2 is walking. Due to the hypothesis of a pick face only pick one item, then, in the next period picker 1 must walking and picker 2 maybe walk or pick. Now, the distance between them is $n-1$ pick face. It means that the transition probability to state $n-1_{ww}$ is q, and transition probability to $n-1_{wp}$ is p.

When the state is 0_{wp}, the distance between two pickers is one pick face. At the moment, picker 2 is picking, and picker 1 is walking. Due to the hypothesis of a pick face only pick one item, then, in the next period picker 2 must walking and picker 1 maybe walk or pick. Now, the distance between them is one pick face. It means that the transition probability to state 1_{ww} is q, and transition probability to 1_{pw} is p.

The circumstances of states transfer from $\{0_{pp},0_{pw},0_{wp},0_{ww}\}$ at t period to state $t+1$ as follows:

$$A_1=\begin{array}{c} \\ 0_{pp}\\0_{pw}\\0_{wp}\\0_{ww}\end{array}\begin{array}{c}\begin{array}{cccc}0_{pp} & 0_{pw} & 0_{wp} & 0_{ww}\end{array}\\ \begin{bmatrix}0 & 0.5 & 0.5 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ p^2 & pq & pq & q^2\end{bmatrix}\end{array}, \quad B=\begin{array}{c} \\ 0_{pp}\\0_{pw}\\0_{wp}\\0_{ww}\end{array}\begin{array}{c}\begin{array}{cccc}1_{pp} & 1_{pw} & 1_{wp} & 1_{ww}\end{array}\\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & p & 0 & q\\ 0 & 0 & 0 & 0\end{bmatrix}\end{array}$$

The circumstances of states transfer from $\{1_{pp},1_{pw},1_{wp},1_{ww}\}$ to various states in the next period as follows:

$$C=\begin{array}{c} \\ 1_{pp}\\1_{pw}\\1_{wp}\\1_{ww}\end{array}\begin{array}{c}\begin{array}{cccc}0_{pp} & 0_{pw} & 0_{wp} & 0_{ww}\end{array}\\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & p & q\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\end{array}, \quad A=\begin{array}{c} \\ 1_{pp}\\1_{pw}\\1_{wp}\\1_{ww}\end{array}\begin{array}{c}\begin{array}{cccc}1_{pp} & 1_{pw} & 1_{wp} & 1_{ww}\end{array}\\ \begin{bmatrix}0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ p^2 & pq & pq & q^2\end{bmatrix}\end{array}, \quad B=\begin{array}{c} \\ 1_{pp}\\1_{pw}\\1_{wp}\\1_{ww}\end{array}\begin{array}{c}\begin{array}{cccc}2_{pp} & 2_{pw} & 2_{wp} & 2_{ww}\end{array}\\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & p & 0 & q\\ 0 & 0 & 0 & 0\end{bmatrix}\end{array},$$

The circumstances of states transfer from $\{(n-1)_{pp},(n-1)_{pw},(n-1)_{wp},(n-1)_{ww}\}$ to various states in the next period as follows:

$$B=\begin{array}{c} \\ n-1_{pp}\\n-1_{pw}\\n-1_{wp}\\n-1_{ww}\end{array}\begin{array}{c}\begin{array}{cccc}0_{pp} & 0_{pw} & 0_{wp} & 0_{ww}\end{array}\\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & p & 0 & q\\ 0 & 0 & 0 & 0\end{bmatrix}\end{array}, \quad C=\begin{array}{c} \\ n-1_{pp}\\n-1_{pw}\\n-1_{wp}\\n-1_{ww}\end{array}\begin{array}{c}\begin{array}{cccc}\text{n-2}_{pp} & \text{n-2}_{pw} & \text{n-2}_{wp} & \text{n-2}_{ww}\end{array}\\ \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & p & 0 & q\\ 0 & 0 & 0 & 0\end{bmatrix}\end{array},$$

Other state transition matrices are consistent with A, B, C matrices. Thus, we can obtain the total state transition matrix as below.

$$T_{1:1}^{W}=\begin{bmatrix} A_1 & B & 0 & \cdots & 0 \\ C & A & B & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & C & A & B \\ B & \cdots & \cdots & C & A \end{bmatrix}_{n\times n} \qquad (1)$$

$T_{1:1}^{W}$ is a $n\times n$ matrix. This matrix differs from the state transition matrix of Gue [5] and Pratik[7] because of different state definition.

4.2 Stationary distribution and blocking rates

Seeking stationary distribution $Z_{1:1}^{W}$ of state transition matrix $T_{1:1}^{W}$, namely solving equation $Z_{1:1}^{W}T_{1:1}^{W}=Z_{1:1}^{W}$, the solution is

$$Z_{1:1}^{W}=[\overbrace{\frac{2p^2(1-p)}{2-p},p,p,\frac{2(1-p)}{2-p}}^{d=0},\overbrace{p^2,p,p,1}^{d=1},\cdots,\overbrace{p^2,p,p,1}^{d=n-1}]$$

Markov stationary distribution density of the process is determined by the second-order norm of $\left\|Z_{1:1}^{W}\right\|$. Because the blocking status is 0_{pp}, each pickers' average blocking rate is

$$b_{1:1}^{W}(k=2)=\frac{z_b}{k\sum_j z_j}=\frac{p^2(1-p)}{n(2-p)(p+1)^2-p-p^3} \qquad (2)$$

From the above equation, $b_{1:1}^{W}(k=2)$ is a decreasing function with respect to n. To get the maximum value, seeking first order partial derivative of p for equation (2) and let the first order derivative is zero, that is

$$b_{1:1}^{W\prime}(k=2)=\frac{np^4+p^4-6np^3+2p^3-3np^2-p^2+4np}{\left[n(2-p)(p+1)^2-p-p^3\right]^2}=0 \qquad (3)$$

When $n=2$, we can draw that picking probability p has four solutions 0，3.7650、-1.0846 and 0.6530 making the above equation hold. And the blocking rate achieves maximum when $p=0.6530$ because of $0\le p\le 1$. Take second derivative of the formula (3), which is less than 0, the maximum value is achieved at this time. Therefore, blocking rate reach maximum, when $p=0.6530$.

4.3 Factors analysis of affecting blocking rate

When take different n values, the relationship between picking probability p and blocking rate b is shown in figure 6 and figure7. That selects the cases when the storage area has four kinds of situation of 2, 5, 10, 20 picking faces, respectively. In the graphics, they are denoted with different curves. This figure shows the situation of the change of blocking rate when the size of the picking region changed. It can be seen from figure 2, when the picking region is fixed, with the increase of picking density, the average blocking rate increases to a maximum value and then decrease. When the picking density p is certain, the average blocking rate b decreases with picking face number n increasing.

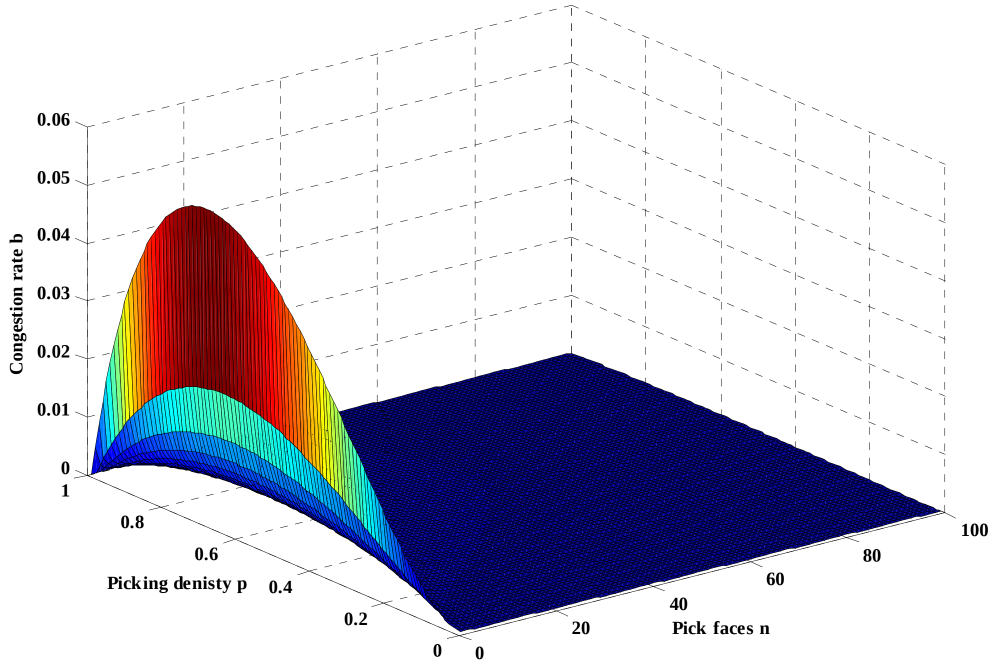


Fig.2 The 3D graph of b、n、p under equal picking and walking speed of single item picking system with wide aisles

5. Conclusions

In this paper, we study the blocking factors affecting the picking efficiency. In the simplest conditions, the picking blocking model is established, That is, in the traditional layout wide aisles system, with equal picking and walking speed, and once only pick one item at a pick face, in this case, we analyze the model. Markov chain method is used to construct the model, deducing one step transition probability matrix, then obtaining the stationary distribution, calculating the analytical expression of the average blocking rate $b_{1:1}^{W}$ with picking probability p and the pick faces number *n*. The matrix of our study differs from the state transition matrix of Gue [5] and Pratik[7] because of different state definition. The relational graph between the picking probability p and the average blocking rate $b_{1:1}^{W}$ is obtained. When the picking and walking speed ratio is $1:1$, there exists a picking probability values which makes maximum blocking rate.

In the following study, we will continue to model and study the complex picking system of more than two pickers, multiple items picking at a pick face, and other speed ratio of picking and walking. Theoretically speaking, our research develops the application of stochastic process theory in storage and picking, lays the foundation for the following study, to provide decision making basis for the efficiency management and optimal control of the picking system, and provide new ideas and methods for the supply chain management theory and the basic operation technology.

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