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Using Finite Element Method for Stability Analysis of Cantilever Retaining Wall

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Abstract

Based on shear strength reduction finite element method, the influence of width of heel plate and toe plate, uniform load, height of retaining wall, parameters of filling on stability of cantilever retaining wall have been analyzed. It is indicated that the fractured surface is the boundary of "protected" and "unprotected" filling soil, which is essentially different with the second fractured surface of planar retaining wall. The stability of retaining wall increased with width of heel plate or toe board or distance of uniform load increased, and decreased with height of retaining wall or value uniform load of increased. It is advised to fill the retaining wall by the soil with low weight, big cohesive strength and big internal friction angle.

Key words: Cantilever retaining wall; heel plate; toe plate; fractured surface; FEM.

1. Introduction

Cantilever retaining wall is a light-duty retaining wall, which keep stability by weight of filling or load above heel plate, characterized with small thickness, light weight, big height and low economic criterion, could be used in the area of lacking building stone and low bearing capacity [1-4].

Cantilever retaining wall composed by vertical wall, heel plate and toe plate. The weight of filling or load above heel plate could enhance the stability of retaining wall, and the toe plate could enhance the factor of safety against overturning. It is easy to make or construct cantilever retaining wall due to its simple shape.

The design of cantilever retaining wall depending on the value of earth pressure act on wall, which could calculated with Rankine's theory or Coulomb's theory. But there are lots of assumptions in Rankine's theory or Coulomb's theory, which is not consistent with reality sometimes, bringing observable errors to value of earth pressure. Expressly, the value of earth pressure has big discreteness with multivariate structure of retaining wall and complex geological conditions.

With the development of finite element method (FEM), it is an effective approach to study stability of retaining wall with FEM. The retaining wall and filling could be treated as a whole in FEM, so the interaction between retaining wall and filling could be considered, the nonlinear earth pressure induced by deformation of retaining wall could also be considered, which agreed with the reality reasonably.

Therefore, based on shear strength reduction of FEM, stability of cantilever retaining wall has been systematic studied in this article, the influence of width of heel plate and toe plate, uniform load, height of retaining wall, parameters of filling on stability of cantilever retaining wall have been analyzed, which aimed to give some advice on design and application of cantilever retaining wall.

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2. Model of Cantilever Retaining Wall with FEM

The height of cantilever retaining wall should not bigger than 6 m, and the strength grade of concrete should not less than C20, the diameter of bar should not less than 12 mm, the embedment depth of foundation should not less than 1.0 m, the filling behind retaining wall should be compacted layered.

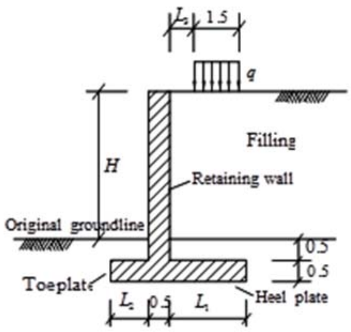


Figure 1 Model of cantilever retaining wall (unit: m)

Now a typical model of cantilever retaining wall is to be analyzed, the thickness of all walls is 0.5 m, the embedment depth of foundation is 1.0 m, the width of heel plate marked as L_1 (m), the width of toe plate marked as L_2 (m), the height of retaining wall marked as H (m). There is an uniform load q (kPa) on top of filling, distance to the back of retaining wall marked as L_3 (m). The geometrical configuration is showed in Fig. 1.

Table 1. Parameters of each layer

Soil layers	Gravity /kN.m^{-3}	Cohesive strength c/kPa	Internal friction angle φ/(^{0})	Deformation Modulus E_0/MPa	Poisson's ratio
Filling	18.6	6.5	18.5	5.8	0.30
Foundation soil Ⅰ	19.5	12.2	25.0	9.1	0.28
Foundation soil Ⅱ	19.9	25.7	28.8	9.9	0.30
Retaining wall	24.2	/	/	2.05E4	0.24

This problem can be treated as plane strain model, foundation and filling could be simulated with Mohr-Coulomb failure criterion, and the retaining wall, constructed with steel concrete with high strength, could be simulated by linear elastic model. The parameters of each layer are showed in table 1.

The friction between retaining wall and soil could be simulated with interface element by parameter R_{inter}. R_{inter}=1.0 indicated that there is no glide between retaining wall and soil. The real value of R_{inter} could be measured by tests, but need much source and fee. In fact, the specific value of every parameter of retaining wall could be impacted by the exact value of R_{inter} from 0 to 1.0, but the regularity of every parameter remain the same with different value of R_{inter}. Therefore, it is assumed that there is no glide between retaining wall and soil with R_{inter}=1.0 in this article.

The domain of FEM model should be large enough to eliminate the influence of boundary. Thus, the area of FEM model including 8 m thickness of foundation, 17 m breadth of filling and 6 m breadth of foundation behind retaining wall. The vertical settlement and lateral displacement fixed at bottom of model, and lateral displacement fixed at both sides of model. The mesh of FEM is divided by 15 nodes triangle elements, as showed in Fig. 2.

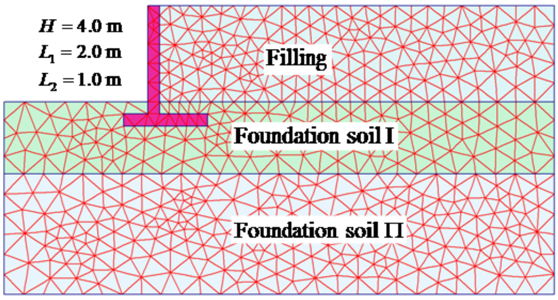


Figure 2. Mesh of finite element method (elements: 1108)

In a general way, the retaining wall is constructed before filling. So, it can be treat that the deformation and consolidation of foundation has been finished at the phase of filling, which simulated by activating the element of filling.

The stability of retaining wall is to be studied with shear strength reduction of FEM [5-8], that is, the intensive parameters c 、 ϕ of each layers soil should be reduced by coefficient F_{trial} simultaneously:

$$c_{\text{r}} = \frac{c}{F_{\text{trial}}}, \quad \phi_{\text{r}} = \arctan\left(\frac{\tan\phi}{F_{\text{trial}}}\right) \tag{1}$$

Where c_{r}, ϕ_{r} is reduced cohesive strength and internal friction angle respectively. The model analyzed by FEM with reduced parameters, if the retaining wall arriving limiting equilibrium state judged by some criterion[9], the safety factor of retaining wall equal the value of coefficient F_{trial}. Otherwise, the model should be recalculated with new reduced parameters until retaining wall arriving limiting equilibrium state. Lots of researches indicated that it is reliably and feasibly to analyze stability of retaining wall with shear strength reduction of FEM [10, 11].

3. Result of fem calculation

3.1 Influence of Width of heel Plate on Stability of Retaining Wall

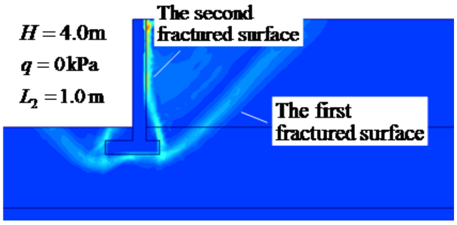
The relationship between width of heel plate and position of fractured surface are showed in Fig. 3. It is obviously that there are two fractured surfaces (the first fractured surface and the second fractured surface) in the filling behind wall, clinging to the bottom of soleplate and cross the soil near toe of wall.

In engineering, retaining wall with the second fractured surface could be called planar retaining wall. The condition of emerging the second fractured surface connected with the slope angle of wall α, friction angle between soil and wall δ, internal friction angle of soil ϕ and slope angle of filling β. In a certain degree, the second fractured surface would be emerged when the slope angle of wall α bigger than the critical slope angle of wall α_{cr} (i.e. $\alpha > \alpha_{\text{cr}}$). The critical slope angle of wall α_{cr} could be calculated as [12]:

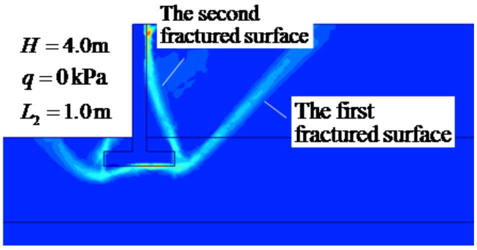
$$\alpha_{\text{cr}} = 45^\circ - \frac{\phi}{2} + \frac{\beta}{2} - \frac{1}{2}\arcsin\left(\frac{\sin\beta}{\sin\phi}\right) \tag{2}$$

When with horizontal filling surface $\beta = 0^\circ$, it can be got $\alpha_{\text{cr}} = 45^\circ - \frac{\phi}{2}$ from equation (2), the corresponding fractured surface showed in Fig. 4.

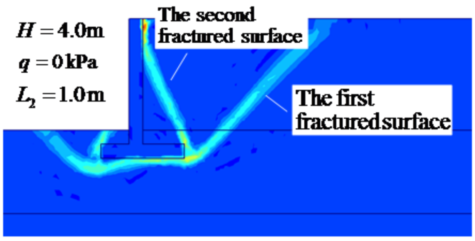
Is that the second fractured surface in Fig. 3 the same as the second fractured surface in Fig. 4? Article [11] holds the attitude that they are the same. In fact, due to the character of structure of cantilever retaining wall, the soil in the triangle zone between vertical wall and heel plate, likely be “protected” by space, is too lower to be failure by shear, as showed in Fig. 5. The volume of “protected” soil increased when width of heel plate increased. That is, the second fractured surface of cantilever retaining wall is the boundary of “protected” and “unprotected” soil.



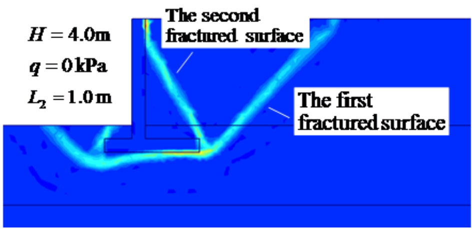
(a) L_1=0.5 m



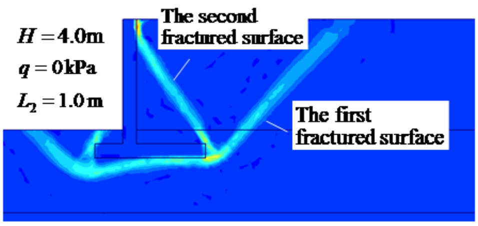
(b) L_1=1.0m



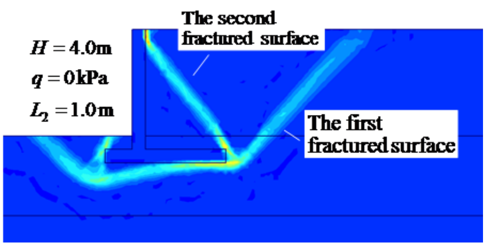
(c) L_1=1.5m



(d) L_1=2.0m



(e) L_1=2.5m



(f) L_1=3.0m

Figure 3. Relationship between width of heel plate and position of fractured surface

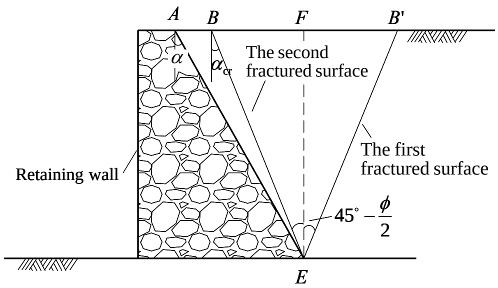


Figure 4. Position of fractured surface of planar retaining wall

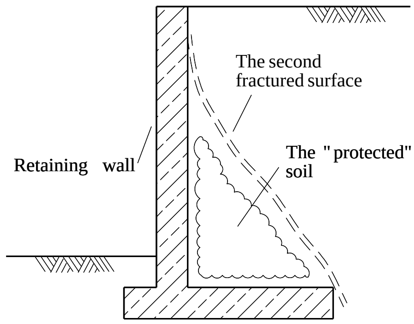


Figure 5. Part of filling apart from fractured surface in cantilever retaining wall

While the typical failure surface of gravity retaining wall, as showed in Fig. 6, is the sliding surface between filling and retaining wall. The result of FEM indicated that the second fractured surface of cantilever retaining wall emerged even at short width of heel plate, such as L_1=0.5 m. Therefore, the second fractured surface of cantilever retaining wall is not the same of that of gravity retaining wall, which is not controlled by equation (2) and $\alpha > \alpha_{cr}$.

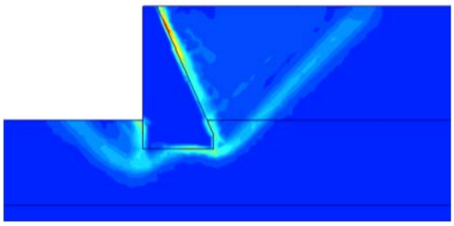


Figure 6. Typical fractured surface of gravity retaining wall

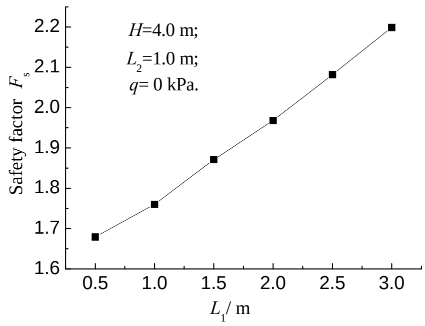
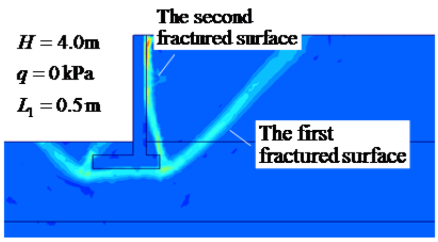


Figure 7. Relationship between safety factor of retaining wall and width of heel plate

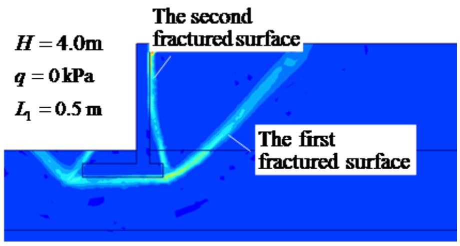
The relationship between safety factor of retaining wall and width of heel plate with the condition of H=4.0 m, L_2=1.0 m, q=0 kPa is showed in Fig. 7. It can be found that the stability of retaining wall increased with the width of heel plate increased. In a certain degree, it is difficult to construct retaining wall with large width of heel plate. Therefore, it needs to choose a reasonable value of width of heel plate according the requirement of real engineering.

3.2 Influence of Width of Toe Plate on Stability of Retaining Wall

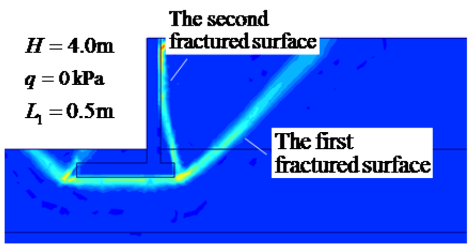
The relationship between width of toe plate and position of fractured surface with the condition of H=4.0 m, L_1=0.5 m, q=0 kPa are showed in Fig. 8. It is could be found that there are no obviously influence on the first fractured surface and the second fractured surface with different width of toe plate, only enlarged the length of fractured surface below toe plate.



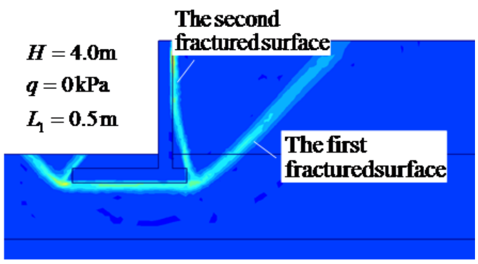
(a) L_2=1.5 m



(b) L_2=2.0 m



(c) L_2=2.5 m



(d) L_2=3.0 m

Figure 8. Relationship between width of toe plate and position of fractured surface

The relationship between safety factor of retaining wall and width of toe plate is showed in Fig. 9. It can be found that the stability of retaining wall increased with the width of toe plate increased. As mentioned above, it is difficult to construct retaining wall with large width of toe plate. Thus it needs to choose a reasonable value of width of heel plate and toe plate according the requirement of real engineering.

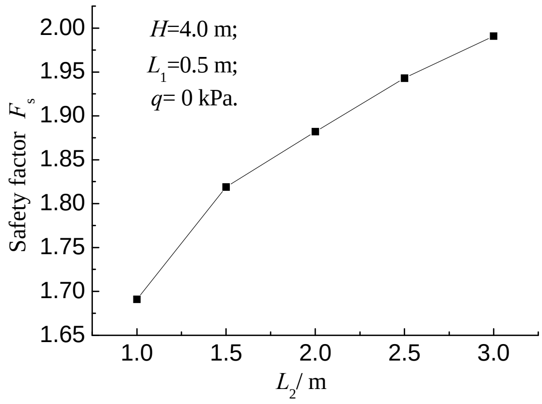
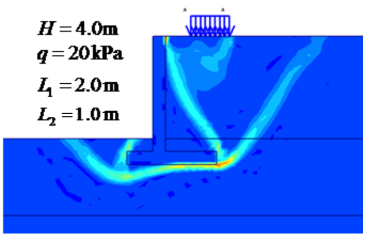


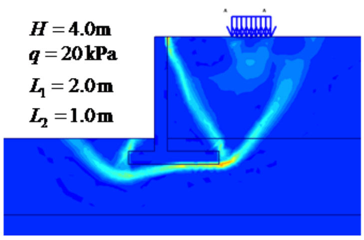
Figure 9. Relationship between safety factor of retaining wall and width of toe plate

3.3 Influence of Uniform Load on Stability of Retaining Wall

The fractured surface of cantilever retaining wall with uniform load at the condition of H=4.0 m, L_1=2.0 m, L_2=1.0 m, q=20 kPa are showed in Fig. 10. It can be found there is no obviously influence on the shape of fractured surface by uniform load.



(a) L_3=1.0 m



(b) L_3=2.5 m

Figure 10. Relationship between location of uniform load and position of fractured surface

The influences of uniform load on stability of cantilever retaining wall are showed in Fig. 11 and Fig. 12. The stability of cantilever retaining wall increased with the distance of uniform load to the back of wall increased, while which decreased with the value of uniform load increased. In order to keep the stability of cantilever retaining wall, uniform load should be applied far form retaining wall, and the value of uniform load should be reduced as possibly.

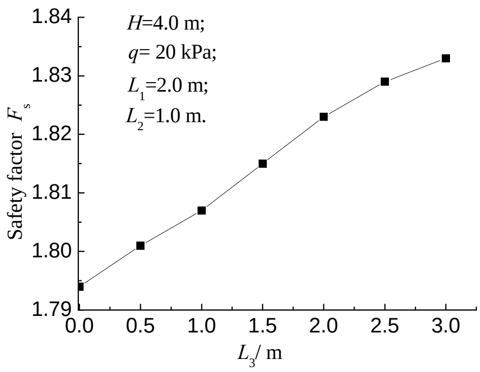


Figure 11. Relationship between safety factor of retaining wall and location of uniform load

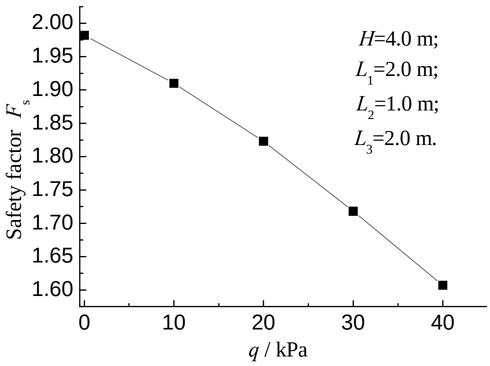
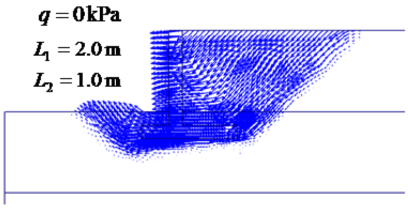
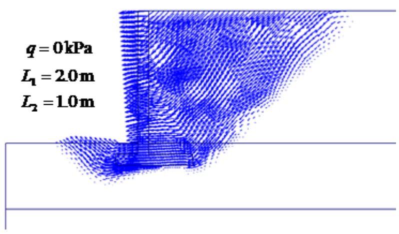


Figure 12. Relationship between safety factor of retaining wall and value of uniform load

3.4 Influence of height of Retaining Wall on its Stability



(a) H=3.0 m



(b) H=6.0 m

Figure 13. Total incremental displacements of filling

The total incremental displacements of filling and stability of cantilever retaining wall influenced by height of retaining wall is showed in Fig. 13 and Fig. 14, respectively. The safety factor of cantilever retaining wall decreased rapidly with height of retaining wall increased. Therefore, it is important to keep a low height of retaining wall as possibly according to real engineering conditions.

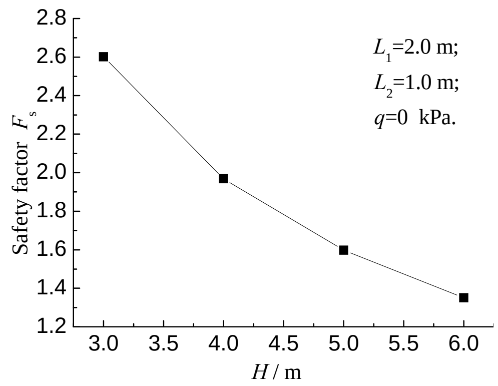


Figure 14 Relationship between safety factor of retaining wall and its height

3.5 Influence of Parameters of Filling on stability of retaining wall

In a certain degree, it is economically to use local materials to fill the retaining wall, such as in mountainous area block stone and reduced stone is to be used, while in plain country clay and sand is to be used. That is, there is big difference in the parameters of different filling, so it is worth to study the influence of parameters of filling on stability of retaining wall.

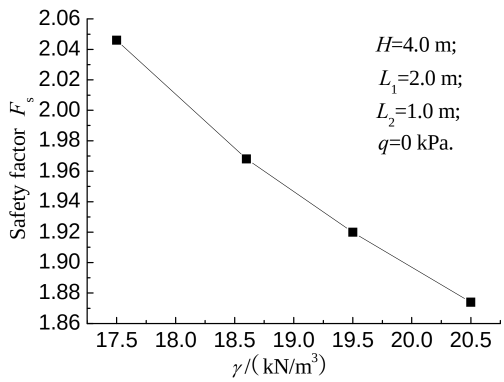


Figure 15 Influence of gravity of filling on stability of retaining wall

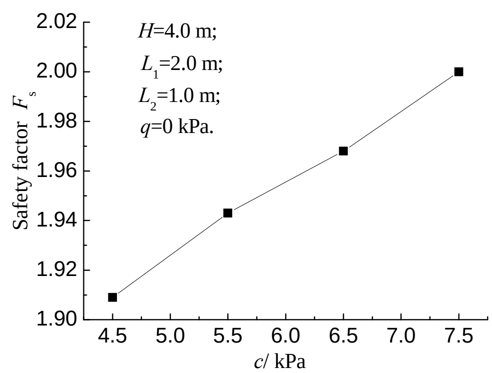


Figure 16 Influence of cohesive strength of filling on stability of retaining wall

The influence of gravity, cohesive strength and inner friction angle of filling on stability of retaining wall are showed in Fig. 15~ Fig. 17. It can be found the stability of retaining wall decreased with gravity of filling increased, which increased with cohesive strength and inner friction angle of filling increased. Therefore, it is suggested to use the filling with small gravity and big cohesive strength and inner friction angle in retaining wall engineering.

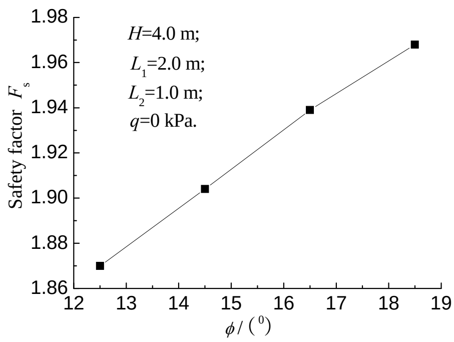


Figure 17 Influence of internal friction angle of filling on stability of retaining wall

Sometimes, expanded polystyrene Sheet (EPS), a material with high strength and very little gravity, could be used to fill retaining wall. The gravity of EPS is 0.2~0.3kN/m^3, and the lateral pressure coefficient of EPS is about K=0.1, which decreased the earth pressure on retaining wall greatly. Without special machine, it is very quickly and conveniently to fill the retaining wall at complex area. It is suggested to use EPS at the area of complicated geology, stability of retaining wall hard to control, difficult to construct with traditional method, special terrain, and so on [13,14].

4. Conclusions

(1) Due to the structure of cantilever retaining wall, the soil in the corner of triangle area formed by vertical wall and heel plate could be “protected”. So the second fractured surface of retaining wall is the boundary of “protected” and “unprotected” soil, which essentially different with planar retaining wall.

(2) The stability of cantilever retaining wall increased with the width of heel plate and toe plate increased. It needs to choose a reasonable value of width of heel plate and toe plate according the requirement of real engineering.

(3) The stability of cantilever retaining wall increased with the distance of uniform load to the back of wall increased, while which decreased with the value of uniform load increased. There is no obviously influence on the shape of fractured surface by uniform load.

(4) The stability of retaining wall decreased with gravity of filling and height of retaining wall increased, while increased with cohesive strength and inner friction angle of filling increased. It is advised to use the filling with small gravity and big cohesive strength and inner friction angle in retaining wall engineering.

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