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Design IIR Filters Using CPSO with Dynamic Varying Search Areas and Lévy Flights

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Abstract

As a nonlinear optimization problem with constraint conditions, the design of an IIR filter is a challenging problem in data/image processing field. In this paper, we use a cooperative quantum particle swarm optimization with dynamic varying search area and Lévy flights (CQPSO-DVSA-LF) algorithm to help design the two-dimensional recursive IIR filters with two ways to reduce the search space. The former is called Dynamic Varying Search Area (DVSA), which takes charge of limiting the ranges of particles' activity into a reduced area. On the other hand, in order to escape the local optima, Lévy flights are used to generate the stochastic disturbance in the movement of particles. From the numerical results of the experiments, we can see that the ripple in the blockage parts of CQPSO-DVSA-LF is smaller than those in other PSO-like algorithms. Moreover, the algorithm is also available and effective in the design of the 2-D elliptic IIR filter with the McClellan transformation method in our study.

Keywords: recursive IIR filter, nonlinear optimization, quantum particle swarm optimization, Lévy flights.

1. Introduction

Recursive and non-recursive filters are widely used in the research and industry domains: seismic data processing, image processing, pattern recognition, remote sensing, etc.[1] A non-recursive FIR filter is one whose impulse response is of finite duration, and output is calculated solely from the current and previous input values. A FIR filter is always stable from its definition, own linear phase characteristic on a wide frequency range, and generally easier to implement. On the other hand, an IIR filter could provide flat frequency response with lower order, and be applied to very narrow transition band frequency. Hence, IIR filters are much more efficient and accurate than FIR filters [2].

Because it is difficult to obtain linear phase response and to control the overall frequency response with IIR filters, the design of an IIR filter is a challenging problem. In the digital 2-D filter design field, there exist principally two different approaches: transformation approach and optimization method. The former needs more pre-knowledge and shows poor performance in most cases, while the later method gains a huge success as it's more simple and efficient [3,4].

Shubhendu Kumar Sarangi et al. presents design of 1-D and 2-D recursive filters using crossover bacterial foraging (COBFO) and Cuckoo Search (CS) techniques, and gain a high quality in [1]; In paper [2], S.K. Saha et al. presents a global heuristic search optimization technique, which is a hybridized version of the Gravitational Search Algorithm (GSA) and Wavelet Mutation (WM) strategy. An extensive simulation study of low-pass (LP), high-pass (HP), band-pass (BP) and band-stop (BS) IIR filters unleashes the potential of GSAWM in achieving better cut-off frequency sharpness, smaller pass band and

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stop band ripples, smaller transition width and higher stop band attenuation with assured stability; Yu Wang, Bin Li, and Thomas Weise, proposed a Two-stage ensemble memetic algorithm for function optimization and digital IIR filter design in their research paper [3]; Jinn-Tsong Tsai et al. emphasizes solving a design problem of two-dimensional (2-D) infinite-impulse-response (IIR) digital structure-specified filters by using an improved genetic algorithm, which is called the hybrid Taguchi-genetic algorithm (HTGA), and then uses the designed filters to the image processing in [4]; Mauricio F. Quélhas presents a new technique for designing IIR filters that have minimum deviation from equiripple response, and the examples are shown to illustrate the efficacy of the proposed design technique, compared to alternative design techniques in [5]; Swagatam Das investigates a novel approach to the designing of two-dimensional zero phase infinite impulse response (IIR) digital filters using the PSO algorithm in his original paper [6]; Adem Kalinli presents a new method for adaptive IIR filter design based on tabu search algorithm in [7].

The contribution of this paper is to propose a new version of PSO algorithm with the global search techniques such as dynamic varying search area and Lévy flights to optimize the design of 2-D IIR filters. The benchmark test experiment show its efficiency and rapid convergence speed than other similar algorithms. Then, the design results show the method yields a better approximation to the transfer function of the IIR filters compared to other method in the state of the art. Furthermore, the method could also support the design of the 2-D elliptic IIR filter with the McClellan transformation method in our study.

The organization of this paper is as follows: Section 1 deals with the introduction part. Formulation of optimized design of 2-D IIR digital filter is presented in Section 2. The proposed CQPSO-DVSA-LF with Dynamic Varying Search Area and Lévy flights is elaborated in Section 3 and 4. Design results are illustrated in Section 5. Concluding remarks are found in the last section.

2. Formulation of optimized design of 2-D IIR digital filter

A system is recursive when the present output always depends on the past/present input and past output of the system. For a 2-D IIR filter with the input-output relationship governed by a form of differential equation as below

$$y(n_1,n_2)=\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\left[b_{ij}y(n_1-i,n_2-j)+a_{ij}x(n_1-i,n_2-j)\right], \tag{1}$$

where $x(n_1,n_2)$ and $y(n_1,n_2)$ are the filter's input and output, respectively; $b_{00}=0$, and N_1,N_2 denote the order on the horizontal and vertical directions respectively. To simplify the problem but without loss of generality, we let $N_1=N_2=N$.

After the z-transformation on the two sides of the equation, the following Eq. 1 could be obtained.

$$Y(n_1,n_2)=\sum_{i=0}^{N}\sum_{j=0}^{N}\left[b_{ij}Y(n_1-i,n_2-j)+a_{ij}X(n_1-i,n_2-j)\right], \tag{2}$$

Hence, the transfer function could be written in the following general form:

$$H(z_1,z_2)=\frac{\sum_{i=0}^{N}\sum_{j=0}^{N}a_{ij}z_1^{-i}z_2^{-j}}{1-\sum_{i=0}^{N}\sum_{j=0}^{N}a_{ij}z_1^{-i}z_2^{-j}} \tag{3}$$

Take the 2-D IIR digital filter referred in papers [x-x] into account, the desired amplitude response characteristic could be formulized as following

$$M_d(\omega_1,\omega_1)=\begin{cases}1, \sqrt[2]{\omega_1^2+\omega_2^2}\le 0.08\pi\\ 0.5, 0.08\pi<\sqrt[2]{\omega_1^2+\omega_2^2}<0.12\pi\\ 0, otherwise\end{cases} \quad (4)$$

where M_d is the desirable amplitude response of 2-D filter of the frequencies ω_1, ω_2 ranged into $[0,\pi]$.

Then, the design task is to find a transfer function $\mathrm{H}(\mathrm{z}_1,\mathrm{z}_2)$ such that it approcimates the desired amplitude response to the greatest extent. Such an approximation of the desired response can be achieved by minimizing

$$J=J\left(a_{ij},q_n,r_n,s_n,H_0\right)=\sum_{i=0}^{N}\sum_{j=0}^{N}[|M(\omega_1,\omega_2)|-M_d(\omega_1,\omega_2)]^p, \quad (5)$$

where $M(\omega_1,\omega_2)$ is the Fourier transform of $H(z_1,z_2)$, that is

$$M(\omega_1,\omega_2)=H(\omega_1,\omega_2)|_{z_2=e^{-j\omega_2}}^{z_1=e^{-j\omega_1}}, \quad (6)$$

and $\omega_1=(\pi/N_1)n_1, \omega_2=(\pi/N_2)n_2$ and p is an even positive integer always equals 2.

Hence, the purpose of design is to minimize the difference between the actual and desired amplitude response of the filter at (N_1,N_2) points which is illustrated by the following equation.

Minimize $J=J\left(a_{ij},q_n,r_n,s_n,H_0\right)$

$$=\sum_{i=0}^{N}\sum_{j=0}^{N}\left[\left|M\left((\pi/N_1)n_1,(\pi/N_2)n_2\right)\right|-M_d\left((\pi/N_1)n_1,(\pi/N_2)n_2\right)\right]^p, \quad (7)$$

For the purpose of discussions, we choose the low order IIR filter as our object, of which stability is easy to judge. Then its transfer function could be written as the below form:

$$H(z_1,z_2)=H_0\frac{\sum_{i=0}^{N}\sum_{j=0}^{N}a_{ij}z_1^iz_2^j}{\prod_{n=1}^{N}(1+q_nz_1+r_nz_2+s_nz_1z_2)}, a_{00}=1 \quad (8)$$

Because of the only first-degree factors into the denominator, the stability conditions are given by

$$|(q_n+r_n)|<1+s_n \quad (9)$$

$$|(q_n-r_n)|<1-s_n \quad (10)$$

$$(1+r_n)>0 \quad (11)$$

$$(1-r_n)>0 \quad (12)$$

which subjects to the constraints

$$|(r_n+s_n)|<q_n+1, \mathrm{n}=1,2,\dots,N \quad (13)$$

where

$$A_R=a_{00}+a_{01}f_{01}+a_{02}f_{02}+a_{10}f_{10}+a_{20}f_{20}+a_{11}f_{11}+a_{12}f_{12}+a_{21}f_{21}+a_{22}f_{22} \quad (14)$$

$$A_I=a_{00}+a_{01}g_{01}+a_{02}g_{02}+a_{10}g_{10}+a_{20}g_{20}+a_{11}g_{11}+a_{12}g_{12}+a_{21}g_{21}+a_{22}g_{22} \quad (15)$$

$$B_{1R}=1+q_1f_{10}+r_1f_{01}+s_1f_{11} \quad (16)$$

$$B_{1I}=q_1g_{10}+r_1g_{01}+s_1g_{11} \quad (17)$$

$$B_{2R}=1+q_2f_{10}+r_2f_{01}+s_2f_{11} \quad (18)$$

$$B_{2I}=q_2g_{10}+r_2g_{01}+s_2g_{11} \quad (19)$$

$$f_{ij}=cos(i\omega_1+i\omega_2), g_{ij}=sin(i\omega_1+i\omega_2), i,j=0,1,2 \quad (20)$$

Therefore, the module amplitude response could be calculated as

$$|M(\omega_1+\omega_2)| = H_0\sqrt{\frac{(A_R^2+A_1^2)}{(B_{1R}^2+B_{1I}^2)(B_{2R}^2+B_{2I}^2)}} \tag{21}$$

3. CQPSO with Dynamic Varying Search Area (DVSA) and Lévy flights

3.1 Rationale of Dynamic Varying Search Area (DVSA)

As we known, complexity of optimization problem is not only relies heavily on the objective/constraint function, but also related with its search area. Simply speaking, subjected to the same objective/constraint function, the larger search area is, the harder it can find the solution [8]. Based on this idea, to change the search area dynamically, or say it reduces, is necessary to accelerate the processing of algorithm. On the other hand, when the search area reduced, the populations of sub-swarms are unnecessary as big as previous.

Given an optimization function:

$$min\, f(x), x = \left(x_1, x_2, \dots, x_{N_d}\right)^T \in S \subseteq R^{N_d} \tag{22}$$

where $S = [a_1, b_1] \times [a_2, b_2] \times \dots \times \left[a_{N_d}, b_{N_d}\right]$, the basic rationale of Dynamic Varying Search Area (DVSA) could be illustrated as the following description: Firstly, assume that N_p cooperative sub-swarms probe in the search space. When the minimal distances between optimal individuals of each sub-swarm reached a threshold, according to the maximum likelihood estimation, the hypothesis that the real optimal solution is in the area arounded by these particles was established. Then reduce the previous search area S to S', generate a new swarm with same sub-swarms on S', and decrease the popula-tions meanwhile. Finally, repeat the above procedures untill satisfy the end condition. Considering the vector *x* before the *r*-th reduce, where the *i*-th component x_i ranges over $[a_i^{r-1}, b_i^{r-1}]$. Then *x* could be expressed as $x^{r-1} \in [a_i^{r-1}, b_i^{r-1}]$. Fig.x examplifies the case that four cooperative sub-swarms reduce their search area. First, they probe the solution in S and get the best particles $x_1^*, x_2^*, x_3^*, x_4^*$ which included in S'. So the search area becomes S'. The next time of reduce to S'' is the same procedure.

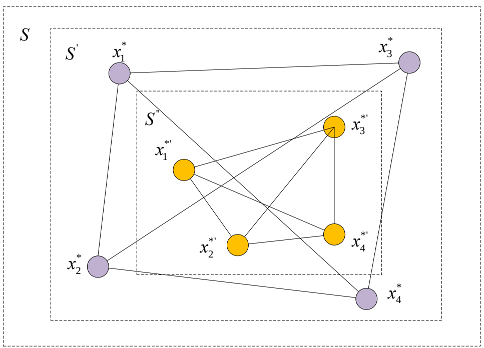


Figure 1. Rationale of DVSA.

3.2 Condition of DVSA

In this part, we will give the condition when the DVSA occures. Surpose there exist N_p sub-swarms, the best particles set found is writen

$$x_b^{r-1} = \{x_b^{r-1,1}, x_b^{r-1,2}, \dots, x_b^{r-1,N_p}\}, \quad x_b^{r-1,p} = (x_{b1}^{r-1,p}, x_{b2}^{r-1,p}, \dots, x_{bN_d}^{r-1,p}), p = 1, 2, \dots, N_p \tag{23}$$

Now, let us consider the distance among them.

$$D(x_b^{r-1,i}, x_b^{r-1,j}) = \left\| x_b^{r-1,i}, x_b^{r-1,j} \right\|_2 \tag{24}$$

$$D^{r-1} = \max_{x_b^{r-1,i}, x_b^{r-1,j} \in x_b^{r-1}} D(x_b^{r-1,i}, x_b^{r-1,j}) \tag{25}$$

where $\left\| \cdot \right\|_2$ is the 2-norm on corresponding search area.

When D^{r-1} reached a small threshold, according to the maximum likelihood estimation, the hypothesis that the real optimal solution is in the area arounded by these particles was established. So the latter search can be performed around these particles.

In light of this, we can give the condition of DVSA as shown in Formula (26).

$$D^{r-1} < \lambda \cdot \left\| a^{r-1} - b^{r-1} \right\|_2, \lambda \in \left(0, 1/N_p \right] \tag{26}$$

In other words, if the above equation is satisfied, then change the search area of the next generation of sub-swarms untill the DVSA occures again. λ can be a fix number, but more often, it is a paramater can be changed adaptively according to the results of evolution.

Let consider the search area after reduce. Note that after the *r*-th reduce, x_i ranges over $[a_i^r, b_i^r]$. Then the upper/Lower bounds are defined by the following equation:

$$\begin{cases} a_i^r = \min\{x_{bi}^{r-1,p}\} - \xi \cdot (b_i^{r-1} - a_i^{r-1}) \\ b_i^r = \min\{x_{bi}^{r-1,p}\} + \xi \cdot (b_i^{r-1} - a_i^{r-1}) \end{cases}, \xi \in \left(0, 1 \right] \tag{27}$$

To guarantee the new search area not larger than the previous area, the above equation should be modified as follows:

$$a_i^r = \begin{cases} a_i^{r-1}, a_i^r < a_i^{r-1} \\ a_i^r, otherwise \end{cases}, b_i^r = \begin{cases} b_i^{r-1}, b_i^r < b_i^{r-1} \\ b_i^r, otherwise \end{cases} \tag{28}$$

3.3 Policy of population scale adjustment

The computational complexity also relies heavily on the scale of the population of the swarm/sub-swarm. In general, the more time about particle evaluation, the more computation takes place. Hence, under the permission of optimization performance, it is necessary to cut down the population of sub-swarms.

In this article, we will follow a traditional method called search granularity. Take the particle after the *r*-th reduce for instance, whose *i*-th component x_i ranges over $[a_i^r, b_i^r]$. The distance of this interval can be written as Eq.(29) which reflect the refined effort of search。If the distance among the solutions is small, we can say that search granularity is small, and vice versa. From the real experience, the bigger swarm, the less distance among the particles, and aslo lessen the search granularity.

$$d_i^r = b_i^r - a_i^r \tag{29}$$

Furthermore, if it is asked that the search granularity on $[a_i^r, b_i^r]$ should be $1/N_{ik}$, the population scale of sub-swarm can be determined abey the below equation.

$$N_i^r = \left\lfloor \prod_{k=1}^{N_d} N_{ik} \cdot d_i^r \right\rfloor \tag{30}$$

where $\lfloor \cdot \rfloor$ is the floor function. When the search area descreses, the population of the related sub-swarm also becomes small.

3.4 Theoretical analysis

In this subsection, an analysis of the convergence of CQPSO with DVSA is provided. We discuss it from two perspectives, i.e., search area and population of swarms.

Firstly, we analyze the varying of interval measure caused by two neighboring reduces. According to the policy of DVSA, it can be described as follows according Eq. (31, 32):

$$b_i^r - a_i^r \le b_i^{r-1} - a_i^{r-1} \tag{31}$$

Without loss of generality, let

$$b_i^r - a_i^r = k_i^r \cdot (b_i^{r-1} - a_i^{r-1}), k_i^r \in (0,1] \tag{32}$$

then

$$\begin{aligned} &b_i^r - a_i^r = k_i^r \cdot (b_i^{r-1} - a_i^{r-1}) = k_i^r \cdot k_i^{r-1}(b_i^{r-2} - a_i^{r-2}) = \ldots = K_i^r \cdot (b_i - a_i),\ K_i^r \\ &= \prod_{j=1}^{r} k_i^r \le \min\{k_i^1, k_i^2, \ldots, k_i^r\} \end{aligned} \tag{33}$$

From the Eq. 33, we can see that when search area varies, the reduced area becomes the k_i^r times of oringin area. So when serveral genenrations of this procedure happens, the final area could be heavily reduced with the considerable promotion of efficiency.

Secondly, in consideration of swarm populations, we can get the result from Eq. 34.

$$N_j^r = \left\lfloor \prod_{i=1}^{N_d} N_{ji} \cdot d_i^r \right\rfloor = \left\lfloor \prod_{j=1}^{N_d} N_{ji} \cdot (b_i^r - a_i^r) \right\rfloor = \left\lfloor \prod_{j=1}^{N_d} N_{ji} \cdot K_i^r (b_i^r - a_i^r) \right\rfloor < \left\lfloor \prod_{j=1}^{N_d} N_{ji} \cdot (b_i - a_i) \right\rfloor \tag{34}$$

The above inference shows that as the search area decreases, the related populations of swarms also be cut down with a certain rate.

4. Lévy flights

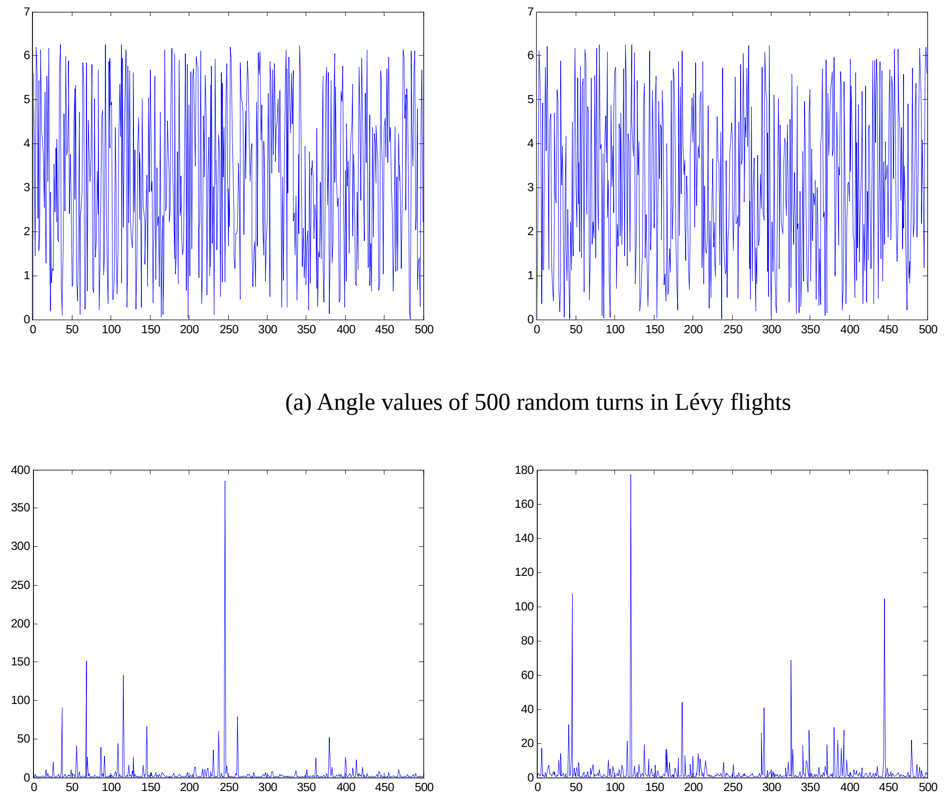
The technique of random disturbance is often imported to improve the performance of PSO or QPSO. When QPSO was proposed, the Gaussian and Cauchy probability distribution disturbance have been used to avoid premature convergence. In [9], the random sequences in QPSO were generated using the absolute value of the Gaussian probability distribution with zero mean and unit variance. Based on the characteristic of QPSO, the variables of the global best and mean best positions are mutated with Cauchy distribution, and an adaptive QPSO version was proposed in [10].

In this paper, another random method, Lévy flights, is employed to do this work. Lévy flights, named after the French mathematician Paul Pierre Lévy, are Markov processes. After a large number of steps, the distance from the origin of the random walk tends to a stable distribution. Lévy flights, which can be characterized by an inverse square distribution of step length, may optimize the random search process when targets are scarce and scarcity of resources. In contrast, Brownian motion is usually suit for the case when need to locate abundant prey or targets. These traits of two random walks inspired us to improve our swarm intelligence optimization, where Lévy flights can improve the ability of “exploration” while

Brownian motion benefits the “exploitation”.

Mathematically, Lévy flights are a kind of random walk whose step lengths meet a heavy-tailed Lévy alpha-stable distribution, often in terms of a power-law formula, $L(s) \sim |s|^{-1-\beta}$, where 0<β≤2 is an index. A typical version of Lévy distribution can be defined as [11].

$$L(s,\gamma,\mu) = \begin{cases} \sqrt{\frac{\gamma}{2\pi}} exp[-\frac{\gamma}{2(s-\mu)}]\frac{1}{(s-\mu)^{3/2}}, \\ \qquad 0<\mu<s<\infty; \\ 0, s \le 0. \end{cases} \tag{35}$$



(a) Angle values of 500 random turns in Lévy flights

(b) Step lengths of 500 random walks in Lévy flights;

Figure 2. 2D Lévy flights in 500 steps

As the change of β, this can evolve into one of Lévy distribution, normal distribution and Cauchy distribution. Taking the 2-D Lévy flights for instance, the steps following a Lévy distribution as in Fig.4(b), while the directions of its movements meet a uniform distribution as in Fig.4(a). Note that the Lévy flights are often efficient in exploring unknown and large-scale search space than Brownian walks. One reason for this argument is that the variance of Lévy flights $\delta^2(t) \sim t^{3-\beta}, 1 \le \beta \le 2$ increases faster than that of Brownian random walks, i.e., $\delta^2(t) \sim t$. Also, compared to Gaussian distribution, Lévy distribution is advantageous since the probability of returning to a previously visited site is smaller than for a Gaussian distribution, irrespective of the value of μ chosen.

5. Proposed algorithm - CQPSO-DVSA-LFD

5.1 Particle representation

For the sake of applying the algorithm to the design optimization problem formulated in the previous section, we need to represent each trail solution as a particle in a multidimensional search space. The real coding technique is used to solve the optimal design problem of 2-D IIR filters. According to Eq. (36), the dimensionality of the present problem is 15, and each particle has 15 positional coordinates represented by the vector

$$X = (a_{01}, a_{02}, a_{10}, a_{11}, a_{12}, a_{20}, a_{21}, a_{22}, q_1, q_2, r_1, r_2, s_1, s_2, H_0)^T \tag{36}$$

5.2 Position update strategy

From the update strategy of CQPSO-DVSA-LFD, we can draw a conclusion that all particles in CQPSO-DVSA-LFD will converge to a common point, leaving the diversity of the population extremely low and particles stagnated without further search before the iterations is over. To overcome the problem, we exert a disturbance generated by Lévy flights on the mean best position, global best position and electoral best position when the swarm is evolving as shown in the following Eq.(37)-Eq.(39). To the local attractor, the hop steps in Lévy flights promise the random traversal in the search space. However, to the global and electoral best location, they only need a slightly disturbance, i.e., the angles meet a uniform distribution, to exploit the particles nearby.

$$C_d' = C_d + \varepsilon_3 \times Step_{Levy} \tag{37}$$

$$P_{gd}^{best\prime} = P_{gd}^{best} + \varepsilon_1 \times Angle_{Levy} \tag{38}$$

$$P_{cgd}^{best\prime} = P_{cgd}^{best} + \varepsilon_2 \times Angle_{Levy} \tag{39}$$

where ε_1, ε_2 , ε_3 is a pre-specified parameter, $Step_{Levy}$ is a number in a sequence by Lévy flights, *angle* is the angles of directions in Lévy flights.

5.3 Equation of particle motion

Differently with other similar methods, we use the output parameters of Lévy flights to intervene the position change directly, which can be seen in the Eq.(40) as follow, where $Angle_{Levy}$ and $Step_{Levy}$ are the output parameters of Lévy flights which are random generated, while ε_1, ε_2, ε_3 are the parametric empirical coefficient.

$$P_{id}^{t+1} = \varphi \times P_{id}^{best} + \psi \times \left(P_{gd}^{best} + \varepsilon_1 \times Angle_{Levy}\right) + (1 - \varphi - \psi) \times (P_{cgd}^{best} + \varepsilon_2 \times Angle_{Levy}) \pm \beta \times \left|(C_d + \varepsilon_3 \times Step_{Levy}) - P_{id}^{t}\right| \times \ln(1/u) \tag{40}$$

5.4 Steps of proposed approach

The CQPSO-DVSA-LF algorithm is a method combing the DVSA with the Lévy flights disturbance technique. Based on the above introduction, we can now present the proposed CQPSO-DVSA-LF algorithm in the following steps in Fig. 3:

Algorithm 1: CQPSO-DVSA-LF
Initiation;
Label 1: Generation primitive sub-swarms;
Foreach sub-swarm-i **In** sub-swarms **Do**
  Calculate the fitness value;
  If (run==first-time)
  Then Update the personal and global optimal position as in QPSO;
  Else Update the personal and global optimal position with;
  Calculate the best particles;
  Check the condition of DVSA, if not satisfied, and then go to the second step.

Calculate the reduced search area.
End Foreach
Calculate the fitness value;
Foreach dimension-i **In** D **Do**
Update the personal and global optimal position;
Update the particles based on quantum behavior with the Lévy disturbance;
End Foreach
Calculate the electoral best position;
Test whether satisfy the condition of termination;
If (Meet terminal condition) **Then** ends
Else repeat from **Label 1**;
End If
End.

Figure 3. Pseudocode of CQPSO-DVSA-LF

6. Experimental results

We run the relevant algorithms on the optimal design of a circular symmetric zero-phase low pass filter according to the literature [4]. All the algorithms have been developed from scratch in MATLAB 2014a platform on an Intel(R) Xeon(R) CPU E3-1230 V2 @ 3.3 GHz and 8GB memory in Windows 7 64bit environment.

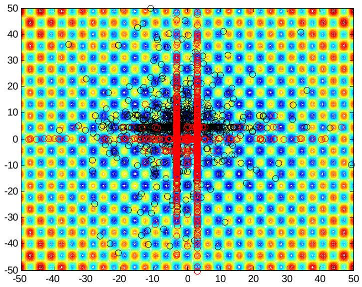
6.1 Preliminary test on function optimization benchmark

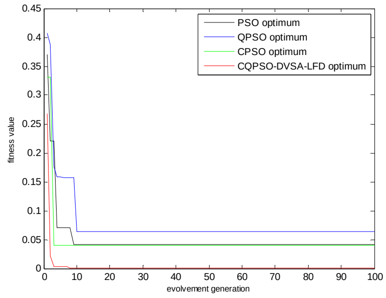
To study the search behavior and its performance of CQPSO-DVSA-LF with other versions of PSO, such as plain PSO, CPSO, and CQPSO, some typical benchmark functions of continuous optimization are selected as the examples [12,13].

Rastrigin's function is frequently used as a test function to test the performance of optimization algorithms. Based on Sphere function, it uses cosine function to generate lots of local optimal points. It is a complex multimodal function, and optimization falls into the local optimum easily. Griewank function is a spin, inseparable variable-dimension multimode function as shown in Fig.4(a). In Fig.4(a), the black cycles denote the distribution of particles of 2-D Griewank function in QPSO under DVSA and LF, while the red ones express that of CQPSO-DVSA-LF with only two cooperative sub-swarms. It can be clearly seen that in CQPSO-DVSA-LF, the search area in each generation of iteration is reduced dynamically into the potential rectangles along two red lines on horizontal/vertical directions. In addition, we can also find that the populations of the latter generations has been reduced obviously, which means the lower computational complexity meanwhile. From Fig.4(b), the results of experiments indicated the proposed CQPSO-DVSA-LF can lead to more efficiency and stability than PSO, QPSO, CPSO and CQPSO.

6.2 Results of 2-D IIR digital filter example

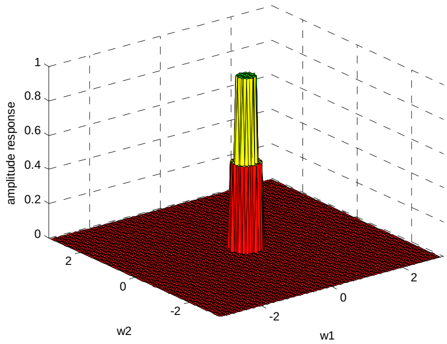
Take the 2-D IIR digital filter referred in papers [2-4] into account, the desired amplitude response characteristic could be formulized as following and Fig. x shows the desired amplitude response |Md(x1, x2)|. Figs. x-x display the frequency responses of the IIR filters designed using NN, PSO, QPSO, CQPSO-DVSA-LF and other competing algorithms. From them, we can see that the ripples in the blockage parts of QPSO and QPSO are smaller than those in other algorithms. The QPSO algorithm is the second contestant herein.

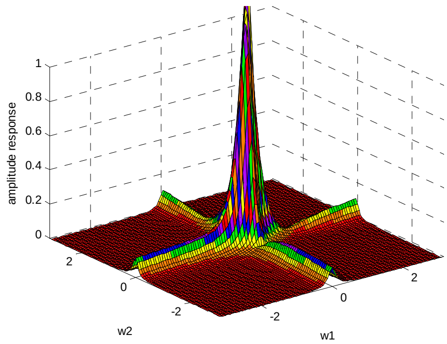




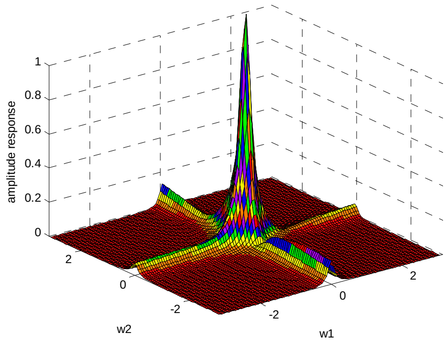
(a). Landscape of Griewank function **(b).** Evolution curves of Griewank function

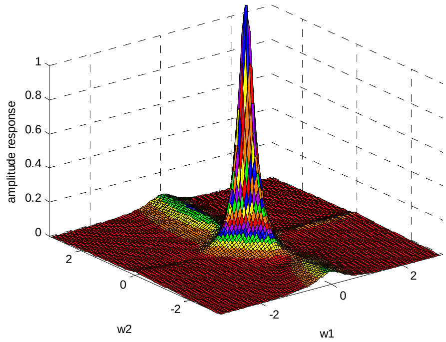
Figure 4. Experimental results on Griewank function



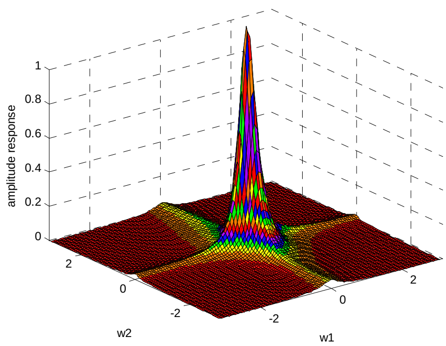


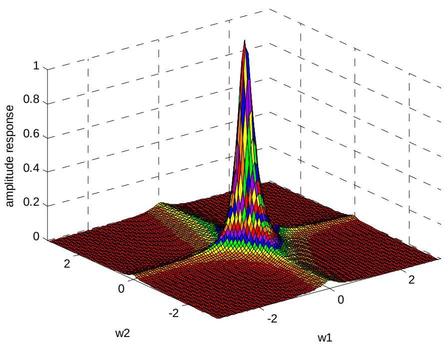
(a). Desired 2-D IIR filter **(b).** 2-D IIR filter by NN





(c). 2-D IIR filter by GA **(d).** 2-D IIR filter by PSO





(e). 2-D IIR filter by QSPO **(f).** 2-D IIR filter by CQPSO-DVSA-LF

Figure 5. Amplitude responses of desired 2-D filter and those by the optimization methods.

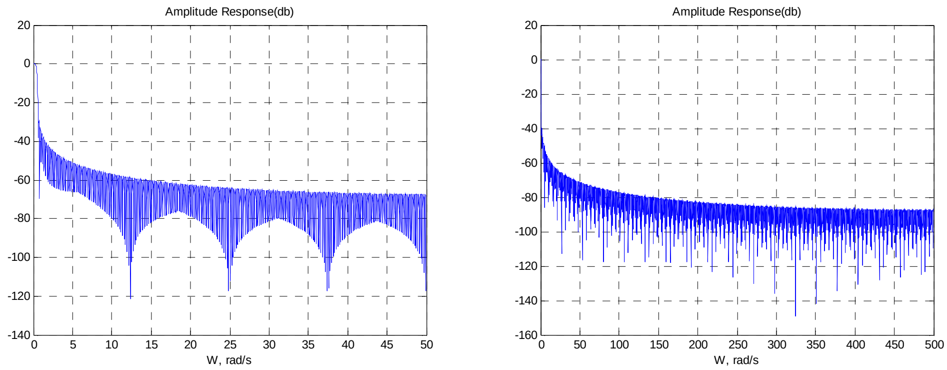


Figure 6. 1-D prototype IIR filter frequency responses

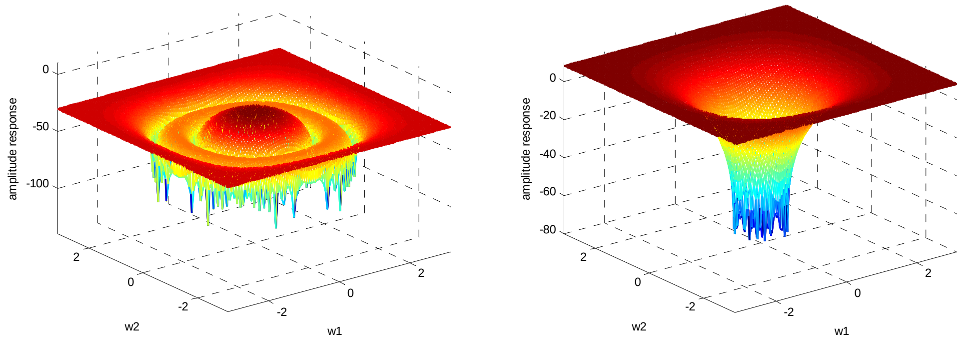


Figure 7. Designed 2-D IIR elliptic filters

Moreover, this method could also be used in some direct design methods. Among the existing methods, the McClellan transformation is a successful one with simplicity and directness. The transformation could obtain a 2-D IIR filter with arbitrary shape by mapping the corresponding 1-D prototype IIR filter frequency samples into 2-D contours. As the coefficients of McClellan transformation are dominated by minimizing the integral squared error along the desired contour, the design problem could also be modeled as a nonlinear optimization problem with constraint conditions. Because of the length of the limit, the concrete formalization could be found in literature [14-15]. In this paper, the proposed method is applied to the design of the 2-D elliptic IIR filter. Considering the 1-D prototype IIR filter frequency responses shown in Fig. 6, the designed 2-D IIR elliptic filters could be obtained as illustrated in Fig. 7.

2. Conclusions

In this paper, we proposed a CQPSO-DVSA-LF algorithm to help design the two-dimensional recursive IIR filters with two ways to reduce the search space. One is called Dynamic Varying Search Area (DVSA), which takes charge of limiting the ranges of particles' activity; the other is cooperative strategy, which divides the candidate solution vector into small sub-swarms. Moreover, to help escape from local optima, a disturbance generated by Lévy flights is embedded as a hybrid strategy. From the numerical results, we can see that the ripple in the blockage parts of CQPSO-DVSA-LF is smaller than those in other PSO-like algorithms, and the QPSO algorithm is the second contestant herein. Moreover, the algorithm is also effective in the design of the 2-D elliptic IIR filter with the McClellan transformation method in our study. Further work will focus on the control of stability about the designed IIR filters.

Acknowledgments

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