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An Approach to Process MIMO Relay Broadcasting Systems with Low-Complexity and Block-Diagonalization

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Abstract

In this paper, we propose two low-complexity block-diagonalization (BD) processing algorithms for multiple-input multiple-output (MIMO) relay broadcasting systems with no channel state information (CSI) at base station. Firstly, a channel inversion and QR decomposition based low-complexity algorithm is introduced to reduce the complexity of the traditional processing algorithm. Then the power loading scheme for the proposed algorithm is analyzed and the closed-form solution is derived. Furthermore, an enhanced algorithm is proposed by employing the minimum-mean-squared-error (MMSE) criterion. Simulation results show that the proposed algorithms reduce the computational complexity significantly. The enhanced algorithm and the power loading scheme improves the sum-rate performance efficiently.

Keywords: MIMO relay broadcasting systems; block diagonalization; low complexity; power loading.

1. Introduction

One major challenge faced by the future cellular wireless communication systems is to provide high data rates for remote users located in the cell boundaries, which experience very low received signal-to-noise ratios (SNRs). An increasingly attractive and cost effective solution is the use of relay stations (RSs). Relays can be classified as full-duplex and half-duplex. Full-duplex relay is still under investigation due to its highly complex hardware implementation. For practical systems, half-duplex relay is more commonly used but suffers significant spectral efficiency loss as a result of the two or more transmission phases needed to deliver a message. Multiple-input multiple-output (MIMO) technique is well known to provide significant improvement of the spectral efficiency and link reliability because of its multiplexing and diversity gains. So combining relay and MIMO techniques can utilize both of their advantages to increase the data rates of remote users.

From a general information theoretic perspective, the capacity bounds of MIMO relay channel with a single user have been analyzed in [1]. For practical implementation, the authors in [2] have investigated the optimal processing matrix at the relay in an amplify-and-forward (AF) relay MIMO system, also with a single user. When multiple antennas are deployed at base station (BS) and RS, multiple users can be scheduled at the same time for broadcasting. However, the processing problem becomes more complex because of the multi-user interference (MUI). For MIMO relay broadcasting systems (MRBS) where each user is equipped with a single antenna, an implementable system architecture was presented in [3] by exploiting the dirty-paper coding (DPC) technique, while the authors in [4] proposed an iterative algorithm for jointly optimizing the precoding matrix at BS and RS to maximize the system capacity. Recently, there has been increasing attention to the MRBS served multiple users equipped with multiple antennas. The studies in [5] proposed a linear processing scheme and the performance with limited feedback channel state information (CSI) was analyzed. To maximize the sum-rate, an iterative algorithm utilizing the

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uplink-downlink duality was developed in [6]. All the above research assumed that BS knows the CSI, which is needed to perform the processing. However, to inform BS of the CSI between RS and users would be rather challenging, especially for high-velocity users. The scenario that only RS has the CSI was considered in [7] and a block-diagonalization (BD) based processing algorithm is employed at RS, which is still too complex to implement in practice.

In this paper, we first extend the work in [7] and propose a low-complexity BD processing algorithm based on channel inversion and QR decomposition (QRD). Then the power loading scheme for the proposed algorithm is analyzed and the closed-form solution is derived. Furthermore, by employing the minimum-mean-squared-error (MMSE) criterion, an enhanced algorithm is introduced to improve the sum-rate.

This paper is organized as follows. The system model is given in Section 2. In Section 3, a brief review of the traditional BD based algorithm is presented. The low-complexity processing algorithms and the corresponding power loading scheme are developed in Section 4. Simulation results and conclusions are displayed in Section 5 and Section 6, respectively.

The following notations are used throughout the paper. Boldface capitals and lowercases denote matrices and vectors. $\boldsymbol{X}^T$, $\boldsymbol{X}^H$, $\boldsymbol{X}^{-1}$ and $\boldsymbol{X}^+$ denote the transpose, conjugate transpose, inverse and pseudo-inverse of $\boldsymbol{X}$, respectively. $\mathbf{I}$ represents the identity matrix.

2. System Model

We consider a MRBS as shown in Figure 1. In this system, the BS with N_T antennas broadcasts independent data streams to K remote users simultaneously and an RS with N_S antennas helps the communication. AF relay is considered due to its simplicity and practicality. The kth user is equipped with N_k antennas and the total number of receive antennas is $N_R = \sum_{k=1}^{K} N_k$. In this paper, we assume $N_T = N_R$ for simplicity. For the case $N_T \neq N_R$, an antenna selection algorithm is needed and the multi-user diversity gain can be exploited. The system configuration can be described as $N_T \times N_S \times \{N_1, \ldots, N_K\}$. We ignore the direct link between BS and each user due to very severe large-scale path loss.

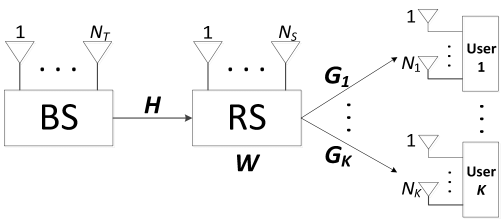

Figure 1. System model of the MRBS

The data transmitted from BS to the kth user is assumed to be an N_k-dimensional symbol vector $\boldsymbol{x}_k$, which is normalized as $E[\boldsymbol{x}_k \boldsymbol{x}_k^H] = \mathbf{I}$. Therefore, the total transmitted data vector at BS can be expressed as $\boldsymbol{x} = [\boldsymbol{x}_1^T, \ldots, \boldsymbol{x}_K^T]^T$. The received data vector at RS is given by

$$\boldsymbol{y}_r = \sqrt{\frac{P_T}{N_T}} \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}_S \tag{1}$$

where P_T is the transmit power at BS and $\boldsymbol{H} \in \mathbb{C}^{N_S \times N_T}$ is the backward channel matrix. $\boldsymbol{n}_S$ is the RS's complex Gaussian noise vector with independent and identically distributed (i.i.d.) elements of zero mean and variance σ_S^2.

After a linear processing, the transmitted data vector at RS is expressed as

$$\boldsymbol{x}_r = \boldsymbol{W}\boldsymbol{y}_r = \sqrt{\frac{P_T}{N_T}}\boldsymbol{W}\boldsymbol{H}\boldsymbol{x} + \boldsymbol{W}\boldsymbol{n}_S \tag{2}$$

where $\boldsymbol{W} \in \mathbb{C}^{N_S \times N_S}$ is the linear processing matrix at RS. The transmit power constraint at RS can be written as

$$tr(\frac{P_T}{N_T}\boldsymbol{W}\boldsymbol{H}\boldsymbol{H}^H\boldsymbol{W}^H + \sigma_S^2\boldsymbol{W}\boldsymbol{W}^H) \le P_S \tag{3}$$

Finally, the received data vector at the kth user is represented as

$$\boldsymbol{x}_k = \boldsymbol{G}_k\boldsymbol{x}_r + \boldsymbol{n}_R = \sqrt{\frac{P_T}{N_T}}\boldsymbol{G}_k\boldsymbol{W}\boldsymbol{H}\boldsymbol{x} + \boldsymbol{G}_k\boldsymbol{W}\boldsymbol{n}_S + \boldsymbol{n}_R \tag{4}$$

where $\boldsymbol{G}_k \in \mathbb{C}^{N_k \times N_S}$ is the forward channel matrix from RS to the kth user and $\boldsymbol{n}_R$ is the user's complex Gaussian noise vector with i.i.d. elements of zero mean and variance σ_R^2. (4) can be rewritten as

$$\boldsymbol{x}_k = \sqrt{\frac{P_T}{N_T}}\boldsymbol{G}_k\boldsymbol{W}\boldsymbol{H}_k\boldsymbol{x}_k + \sqrt{\frac{P_T}{N_T}}\boldsymbol{G}_k\boldsymbol{W}\sum_{j=1, j\ne k}^{K}\boldsymbol{H}_j\boldsymbol{x}_j + \boldsymbol{G}_k\boldsymbol{W}\boldsymbol{n}_S + \boldsymbol{n}_R \tag{5}$$

where $\boldsymbol{H}_k \in \mathbb{C}^{N_S \times N_k}$ represents the channel matrix from the specific N_k transmit antennas at BS which are dedicated to the kth user, to the N_S receive antennas at RS.

As we can see from (5), the first term of the right hand side indicates the desired data of the kth user and the others includes the MUI and noise. To fully eliminate the MUI in (5), $\boldsymbol{W}$ should satisfy

$$\boldsymbol{G}_k\boldsymbol{W}\boldsymbol{H}_j \begin{cases} \ne 0, & \text{for } j = k \\ = 0, & \text{otherwise} \end{cases} \tag{6}$$

or

$$\boldsymbol{G}\boldsymbol{W}\boldsymbol{H} = \begin{bmatrix} \boldsymbol{G}_1\boldsymbol{W}\boldsymbol{H}_1 & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & \boldsymbol{G}_K\boldsymbol{W}\boldsymbol{H}_K \end{bmatrix} \tag{7}$$

where $\boldsymbol{G} = [\boldsymbol{G}_1^T \ ... \ \boldsymbol{G}_K^T]^{\mathrm{T}}$ is the combining forward channel matrix. We define $\boldsymbol{W}$ as

$$\boldsymbol{W} = \boldsymbol{W}_G\boldsymbol{W}_H \tag{8}$$

$\boldsymbol{W}_G$ and $\boldsymbol{W}_H$ are the linear processing matrices for the forward channel and the backward channel, respectively. It is obvious that (7) is satisfied when we set $\boldsymbol{W}_G = \boldsymbol{G}^+$ and $\boldsymbol{W}_H = \boldsymbol{H}^+$. However, this scheme simply treats each antenna as a single user thus sacrificing some benefit of the multiple antennas.

3. Review of the BD Based Algorithm

The authors in [7] proposed a BD based algorithm, where $\boldsymbol{W}_G$ and $\boldsymbol{W}_H$ are designed to place each user's data stream at the null space of the other users' channels.

To this end, the kth user's forward MUI channel matrix is defined as

$$\overline{\boldsymbol{G}}_k = [\boldsymbol{G}_1^T \ ... \ \boldsymbol{G}_{k-1}^T \ \boldsymbol{G}_{k+1}^T \ ... \ \boldsymbol{G}_K^T]^T \in \mathbb{C}^{\overline{N}_{Gk} \times N_S} \tag{9}$$

where $\overline{N}_{Gk} = N_R - N_k$. Assuming $N_S \ge N_R$, we have $\mathrm{rank}(\overline{\boldsymbol{G}}_k) = \overline{N}_{Gk}$ in a rich scattering environment. The singular value decomposition (SVD) of $\overline{\boldsymbol{G}}_k$ is

$$\overline{\boldsymbol{G}}_k = \boldsymbol{U}_{Gk}\boldsymbol{\Sigma}_{Gk}[\boldsymbol{V}_{Gk,1}\ \boldsymbol{V}_{Gk,0}]^{\mathrm{H}} \tag{10}$$

where $\boldsymbol{U}_{Gk} \in \mathbb{C}^{\overline{N}_{Gk}\times\overline{N}_{Gk}}$ is a unitary matrix and the diagonal matrix $\boldsymbol{\Sigma}_{Gk} \in \mathbb{C}^{\overline{N}_{Gk}\times N_S}$ contains the singular values of $\overline{\boldsymbol{G}}_k$. $\boldsymbol{V}_{Gk,1} \in \mathbb{C}^{N_S\times\overline{N}_{Gk}}$ consists of the first $\overline{N}_{Gk}$ non-zero singular vectors and $\boldsymbol{V}_{Gk,0} \in \mathbb{C}^{N_S\times(N_S-\overline{N}_{Gk})}$ holds the last $N_S-\overline{N}_{Gk}$ zero singular vectors. Thus, $\boldsymbol{V}_{Gk,0}$ forms an orthonormal basis for the null space of $\overline{\boldsymbol{G}}_k$ and $\boldsymbol{W}_G$ can be expressed as

$$\boldsymbol{W}_G = [\boldsymbol{V}_{G1,0}\ \dots\ \boldsymbol{V}_{GK,0}] \tag{11}$$

Similarly, the kth user's backward MUI channel matrix is defined as

$$\overline{\boldsymbol{H}}_k = [\boldsymbol{H}_1\ \dots\ \boldsymbol{H}_{k-1}\ \boldsymbol{H}_{k+1}\ \dots\ \boldsymbol{H}_K] \in \mathbb{C}^{N_S\times\overline{N}_{Hk}} \tag{12}$$

where $\overline{N}_{Hk} = N_T - N_k$. The SVD of $\overline{\boldsymbol{H}}_k$ is

$$\overline{\boldsymbol{H}}_k = [\boldsymbol{U}_{Hk,1}\ \boldsymbol{U}_{Hk,0}]\boldsymbol{\Sigma}_{Hk}\boldsymbol{V}_{Hk}^H \tag{13}$$

where $\boldsymbol{U}_{Hk,0}^H \in \mathbb{C}^{(N_S-\overline{N}_{Hk})\times N_S}$ forms an orthonormal basis for the null space of $\overline{\boldsymbol{H}}_k$ and $\boldsymbol{W}_H$ can be expressed as

$$\boldsymbol{W}_H = [\boldsymbol{U}_{H1,0}\ \dots\ \boldsymbol{U}_{HK,0}]^H \tag{14}$$

After the determination of $\boldsymbol{W}_G$ and $\boldsymbol{W}_H$, the MUI at each user is fully eliminated. Since this BD based algorithm uses SVD to calculate the linear processing matrix, we term it as the BD-SVD algorithm.

4. Low-Complexity Processing Algorithms

In this section, we introduce the low-complexity processing algorithms to reduce the complexity and improve the performance of the MRBS.

4.1. Low-Complexity Alogorithm

Although the BD-SVD algorithm can fully eliminate the MUI and benefit from the multi-antenna gain, the SVD operations bring along considerable computational complexity which makes it difficult to implement in practice. In order to reduce the complexity, we propose a BD based low-complexity algorithm (BD-LC) as follows.

We first define the pseudo-inverse of the forward channel matrix $\boldsymbol{G}$ as

$$\boldsymbol{G}^+ = \boldsymbol{G}^H(\boldsymbol{G}\boldsymbol{G}^H)^{-1} = [\boldsymbol{G}_1^+\ \dots\ \boldsymbol{G}_K^+] \tag{15}$$

By performing QRD on $\boldsymbol{G}_k^+$, we get

$$\boldsymbol{G}_k^+ = \boldsymbol{Q}_{Gk}\boldsymbol{R}_{Gk},\quad k=1,\ \dots,\ K \tag{16}$$

where $\boldsymbol{Q}_{Gk} \in \mathbb{C}^{N_S\times N_k}$ forms an orthonormal basis for the column space of $\boldsymbol{G}_k^+$ and $\boldsymbol{R}_{Gk} \in \mathbb{C}^{N_k\times N_k}$ is an upper triangular matrix[8]. It is observed that $\boldsymbol{G}_j\boldsymbol{G}_k^+ = \boldsymbol{G}_j\boldsymbol{Q}_{Gk}\boldsymbol{R}_{Gk} = \boldsymbol{0}$, for $j \neq k$. Since $\boldsymbol{R}_{Gk}$ is non-singular, it follows $\boldsymbol{G}_j\boldsymbol{Q}_{Gk} = \boldsymbol{0}$. Therefore, $\boldsymbol{Q}_{Gk}$ forms an orthonormal basis for the null space of $\overline{\boldsymbol{G}}_k$. According to [9], $\boldsymbol{Q}_{Gk}$ is equivalent to the $\boldsymbol{V}_{Gk,0}$ in the BD-SVD algorithm. Therefore, $\boldsymbol{W}_G$ can be expressed as

$$\boldsymbol{W}_G = [\boldsymbol{Q}_{G1}\ \dots\ \boldsymbol{Q}_{GK}] \tag{17}$$

$\boldsymbol{W}_H$ can be calculated in a similar way. We define

$$\boldsymbol{H}^+ = (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H = [\boldsymbol{H}_1^+\ \dots\ \boldsymbol{H}_K^+]^H \tag{18}$$

By performing QRD operation on $\boldsymbol{H}_k^+$, we get

$$\boldsymbol{H}_k^+ = \boldsymbol{Q}_{Hk}\boldsymbol{R}_{Hk},\ k=1,\ \ldots,\ K \tag{19}$$

where $\boldsymbol{Q}_{Hk} \in \mathbb{C}^{N_S \times N_k}$ forms an orthonormal basis for the null space of $\overline{\boldsymbol{H}}_k$ and $\boldsymbol{W}_H$ can be expressed as

$$\boldsymbol{W}_H = [\boldsymbol{Q}_{H1}\ \ldots\ \boldsymbol{Q}_{HK}]^H \tag{20}$$

4.2. Complexity Analysis

We use the number of floating point operations (FLOPs) to measure the computational complexity. According to [9], the numbers of FLOPs required for different matrix operations are summarized as follows:

- Multiplication of $m \times n$ and $n \times p$ complex matrices: $8mnp$;
- SVD of an $m \times n$ ($m \le n$) complex matrix where only $\boldsymbol{\Sigma}$ and $\boldsymbol{V}$ are obtained: $32(nm^2 + 2m^3)$;
- Inversion of an $m \times m$ real matrix: $16m^3/3$;
- QRD of an $m \times n$ ($m \le n$) complex matrix: $16(n^2m - nm^2 + 1/3m^3)$.

The required numbers of FLOPs for BD-SVD and BD-LC are illustrated in Table 1 and Table 2, respectively.

Table 1. Complexity of BD-SVD

Steps	Operations	FLOPs
1	SVD for calculating $\boldsymbol{W}_G$	$32\sum_{k=1}^{K}(N_S\overline{N}_{Gk}^2 + 2\overline{N}_{Gk}^3)$
2	SVD for calculating $\boldsymbol{W}_H$	$32\sum_{k=1}^{K}(N_S\overline{N}_{Hk}^2 + 2\overline{N}_{Hk}^3)$

Table 2. Complexity of BD-LC

Steps	Operations	FLOPs
1	Inversion of $\boldsymbol{G}$	$16/3N_R^3 + 16N_R^2N_S$
2	QRD for calculating $\boldsymbol{W}_G$	$16\sum_{k=1}^{K}(N_S^2N_k - N_SN_k^2 + 1/3N_k^3)$
3	Inversion of $\boldsymbol{H}$	$16/3N_T^3 + 16N_T^2N_S$
4	QRD for calculating $\boldsymbol{W}_H$	$16\sum_{k=1}^{K}(N_S^2N_k - N_SN_k^2 + 1/3N_k^3)$

In order to make the complexity comparison more comprehensive and intuitive, we plot the numbers of FLOPs required for the two algorithms in Figure 2 as a function of user number. We assume that each user is equipped with $N_k = 2$ antennas and $N_T = N_S = N_k \times K$.

It can be seen from Figure 2 that the proposed BD-LC algorithm demands much lower computational complexity than the BD-SVD algorithm. One reason is that the QRD operation is much simpler than the SVD operation in the case of same matrix dimension. A more important reason is that, the SVD operations in the BD-SVD algorithm are implemented K times on matrices with dimensions $\overline{N}_{Gk} \times N_S$ and $N_S \times \overline{N}_{Hk}$, while the QRD operations in the BD-LC algorithm are implemented K times on matrices with dimensions $N_S \times N_k$ and $N_S \times N_k$, which are much lower than the formers. It is worth noting that with the increase of the system dimension, the complexity reduction by the proposed BD-LC algorithm becomes more considerable.

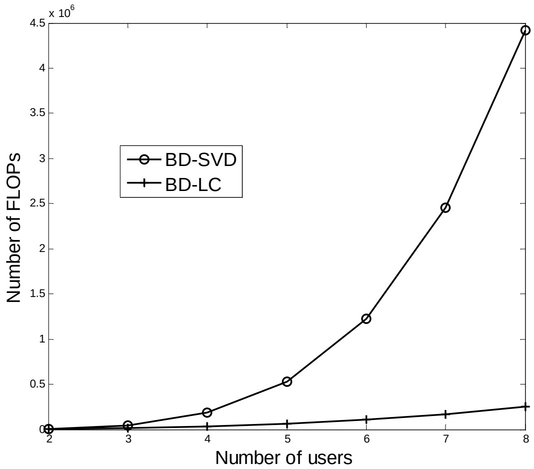

Figure 2. Complexity comparison, with $N_k = 2$ and $N_T = N_S = N_k \times K$

4.3. Power Loading Scheme

To maximize the sum-rate, we define the power loading matrix $\boldsymbol{P}$ as

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_1 & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & \boldsymbol{P}_K \end{bmatrix} \tag{21}$$

where $\boldsymbol{P}_k \in \mathbb{C}^{N_k \times N_k}$ is the power loading matrix for the kth user. With the power loading matrix being introduced, (8) can be rewritten as

$$\boldsymbol{W} = \boldsymbol{W}_G \boldsymbol{P} \boldsymbol{W}_H \tag{22}$$

By substituting (17), (20), (21) and (22) into (5), we get

$$\boldsymbol{y}_k = \sqrt{\frac{P_T}{N_T}} \widetilde{\boldsymbol{G}}_k \boldsymbol{P}_k \widetilde{\boldsymbol{H}}_k \boldsymbol{x}_k + \widetilde{\boldsymbol{G}}_k \boldsymbol{P}_k \boldsymbol{Q}_{Hk}^H \boldsymbol{n}_S + \boldsymbol{n}_R \tag{23}$$

where $\widetilde{\boldsymbol{G}}_k = \boldsymbol{G}_k \boldsymbol{Q}_{Gk}$ and $\widetilde{\boldsymbol{H}}_k = \boldsymbol{Q}_{Hk}^H \boldsymbol{H}_k$ denote the effective forward and backward channel matrix, respectively. As we can see, after the processing, the MRBS becomes K parallel single-user MIMO relay systems. Motivated by the optimal design for the single-user MIMO relay system[2], we perform SVD operations on $\widetilde{\boldsymbol{G}}_k$ and $\widetilde{\boldsymbol{H}}_k$ as $\widetilde{\boldsymbol{G}}_k = \widetilde{\boldsymbol{U}}_{GK} \widetilde{\boldsymbol{\Sigma}}_{GK} \widetilde{\boldsymbol{V}}_{GK}^H$ and $\widetilde{\boldsymbol{H}}_k = \widetilde{\boldsymbol{U}}_{HK} \widetilde{\boldsymbol{\Sigma}}_{HK} \widetilde{\boldsymbol{V}}_{HK}^H$. Then $\boldsymbol{P}_k$ is formulated as

$$\boldsymbol{P}_k = \widetilde{\boldsymbol{V}}_{Gk} \boldsymbol{\Lambda}_k \widetilde{\boldsymbol{U}}_{Hk}^H \tag{24}$$

where $\Lambda_k = \mathrm{diag}(\sqrt{\Lambda_{k,1}}, ..., \sqrt{\Lambda_{k,N_k}})$ is for allocating the power. By substituting (24) into (23), we get

$$\boldsymbol{y}_k = \sqrt{\frac{P_T}{N_T}} \widetilde{\boldsymbol{U}}_{GK} \widetilde{\boldsymbol{\Sigma}}_{GK} \boldsymbol{\Lambda}_k \widetilde{\boldsymbol{\Sigma}}_{HK} \widetilde{\boldsymbol{V}}_{HK}^H \boldsymbol{x}_k + \widetilde{\boldsymbol{U}}_{GK} \widetilde{\boldsymbol{\Sigma}}_{GK} \boldsymbol{\Lambda}_k \widetilde{\boldsymbol{U}}_{Hk}^H \boldsymbol{Q}_{Hk}^H \boldsymbol{n}_S + \boldsymbol{n}_R \tag{25}$$

The sum-rate at the kth user is derived as

$$
\begin{aligned}
R_k &= \frac{1}{2}\log_2 \det(\mathbf{I} + \frac{\frac{P_T}{N_T}\tilde{\Sigma}_{HK}^2 \Lambda_k^2 \tilde{\Sigma}_{GK}^2}{\sigma_S^2 \Lambda_k^2 \tilde{\Sigma}_{GK}^2 + \sigma_R^2}) \\
&= \frac{1}{2}\sum_{j=1}^{N_k}\log_2(1 + \frac{\frac{P_T}{N_T}\tilde{\Sigma}_{Hk,j}\Lambda_{k,j}\tilde{\Sigma}_{Gk,j}}{\sigma_S^2 \Lambda_{k,j}\tilde{\Sigma}_{Gk,j} + \sigma_R^2})
\end{aligned} \tag{26}
$$

where the factor of 1/2 comes from the loss of the half-duplex transmission. $\tilde{\Sigma}_{Hk,j}$ and $\tilde{\Sigma}_{Gk,j}$ are the jth elements of the diagonal matrix $\tilde{\Sigma}_{HK}^2$ and $\tilde{\Sigma}_{GK}^2$, respectively.

The transmit power constraint can be written as

$$
\sum_{k=1}^{K} tr(\frac{P_T}{N_T}\Lambda_k^2 \tilde{\Sigma}_{Hk}^2 + \sigma_S^2 \Lambda_k^2) = \sum_{k=1}^{K}\sum_{j=1}^{N_k}\Lambda_{k,j}(\frac{P_T}{N_T}\tilde{\Sigma}_{Hk,j} + \sigma_S^2) \le P_S \tag{27}
$$

Therefore, the maximizing problem of the system sum-rate $R = \sum_{k=1}^{K} R_k$ is formulated as

$$
\begin{aligned}
&\underset{\Lambda_{k,j}}{\text{maxmize}} \;\; R \\
&\text{subject to} \;\; \Lambda_{k,j} \ge 0,\ \forall k,\ j; \\
&\qquad \sum_{k=1}^{K}\sum_{j=1}^{N_k}\Lambda_{k,j}(\frac{P_T}{N_T}\tilde{\Sigma}_{Hk,j} + \sigma_S^2) \le P_S
\end{aligned} \tag{28}
$$

Problem (28) is a standard convex optimization problem and can be solved with the Lagrange multiplier method. The closed-form solution is given as

$$
\Lambda_{k,j} = \frac{\sigma_R^2\left(\sqrt{(\rho\tilde{\Sigma}_{Hk,j})^2 + \frac{4\rho\tilde{\Sigma}_{Hk,j}\tilde{\Sigma}_{Gk,j}\mu}{\ln 2\sigma_R^2}} - \rho\tilde{\Sigma}_{Hk,j} - 2\right)^+}{2\sigma_S^2\tilde{\Sigma}_{Gk,j}(1+\rho\tilde{\Sigma}_{Hk,j})} \tag{29}
$$

where $\rho = P_T/(N_T\sigma_S^2)$ and $(x)^+ = \max(0,x)$. μ is a unique root of $\sum_{k=1}^{K}\sum_{j=1}^{N_k}\Lambda_{k,j}(P_T\tilde{\Sigma}_{Hk,j}/N_T + \sigma_S^2) - P_S = 0$, which can be solved with a numerical root-finding algorithm, such as the Bisection method, etc.

4.4. Enhanced Algorithm

The pseudo-inverses in (15) and (18) may amplify the noise and therefore some performance loss will occur. To overcome this problem, an enhanced algorithm termed the BD based enhanced low-complexity algorithm (BD-ELC) is proposed as follows.

To take the noise term into account, we apply the MMSE criterion and replace the pseudo-inverse in (15) and (18) with MMSE channel inversion as [10]

$$
\boldsymbol{G}_{mmse}^{+} = \boldsymbol{G}^H(\boldsymbol{G}\boldsymbol{G}^H + \alpha\mathbf{I})^{-1} \tag{30}
$$

$$
\boldsymbol{H}_{mmse}^{+} = (\boldsymbol{H}^H\boldsymbol{H} + \beta\mathbf{I})^{-1}\boldsymbol{H}^H \tag{31}
$$

where $\alpha = N_S\sigma_R^2/P_S$ and $\beta = N_T\sigma_S^2/P_T$. The remaining steps are similar to the BD-LC algorithm, which indicates similar computational complexity.

Note that the BD-LC algorithm relies on the pseudo-inverse of $\boldsymbol{G}$ and $\boldsymbol{H}$, thus having a dimension constraint, i.e., $N_S \le N_T$ and $N_R \le N_S$. However, this constraint does not exist for the BD-ELC algorithm, similar to the MMSE channel inversion.

Unlike the BD-LC algorithm, the power loading scheme for the BD-ELC algorithm is not easy to identify since the residual MUI varies according to the power loading. However, the proposed power loading scheme can still be implemented in BD-ELC, and improve the sum-rate performance efficiently as will be shown later.

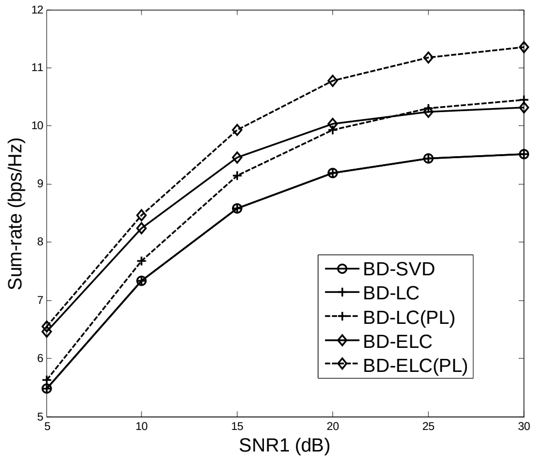

Figure 3. Sum-rate as a function of SNR1 with SNR2 fixed at 10dB.

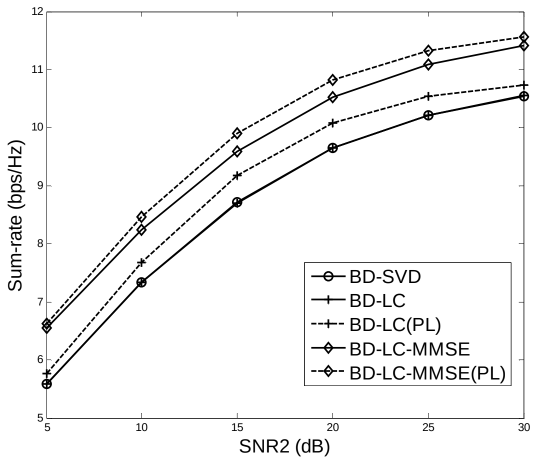

Figure 4. Sum-rate as a function of SNR2 with SNR1 fixed at 10dB.

5. Simulation Results

In this section, we present simulation results to show the sum-rate performance of the proposed algorithms. A system under the configuration of $6 \times 6 \times \{2,2,2\}$ is considered. All channels are assumed to be quasi-static flat faded and the elements are complex Gaussian variables with zero mean and unit variance. The average received SNRs per antenna at RS and at users is denoted as SNR1 and SNR2, respectively. PL in the legend denotes the implementation of the proposed power loading scheme.

Figure 3 shows the sum-rate performance of different algorithms as a function of SNR1 with SNR2 fixed at 10dB, and vice versa in Figure 4. It can be seen that the proposed BD-LC algorithm represents identical performance as the traditional BD-SVD algorithm, while offering much lower computational complexity as analyzed in section IV. The proposed power loading scheme improves the sum-rate performance efficiently, at a cost of additional computational complexity. However, even without the power loading scheme, the BD-ELC algorithm shows significant sum-rate enhancement for balancing the MUI and the noise. The proposed power loading scheme can improve the sum-rate performance of the BD-ELC algorithm as well.

6. Conclusion

In this paper, two generalized BD algorithms for the MRBS are proposed. We first extend the traditional BD based algorithm to a low-complexity BD-LC algorithm, which reduces the computational complexity significantly. Then a power loading scheme for the proposed algorithm is developed to improve the sum-rate performance. By employing the MMSE criterion, an enhanced algorithm is proposed to further improve the performance. Simulation results show that the proposed BD-ELC algorithm is superior to the other algorithms and the power loading scheme improves the sum-rate performance efficiently.

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